Semester 1	Solution to Tutorial Set 6	2013

1. $\mathrm{E}(X) = np = 10 \times 0.3 = 3$, $\mathrm{Var}(X) = np(1-p) = 10 \times 0.3 \times 0.7 = 2.1$ and $\mathrm{SD}(X) = \sqrt{2.1} = 1.4491$.

2. Let $Y \sim \mathrm{B}(15, 0.4)$. Therefore, $\mathrm{P}(X = 13) = \mathrm{P}(Y = 2) = 0.0219$.
   Similarly, $\mathrm{P}(X = 5) = 0.0245$.

3. Answer (d).

4. $X \sim \mathrm{N}(2, 3^2)$. Draw normal curves to illustrate the areas.

   (a) $\mathrm{P}(Z < 1.2) = 0.8849$.
   R command is `pnorm(1.2)`.

   (b) $\mathrm{P}(Z > -1.2) = \mathrm{P}(Z < 1.2) = 0.8849$.
   R command is `1-pnorm(-1.2)` or `pnorm(-1.2,lower.tail = F)`.

   (c) $\mathrm{P}(Z < -1.2) = \mathrm{P}(Z > 1.2) = 1 - \mathrm{P}(Z > 1.2) = 1 - 0.8849 = 0.1151$.
   R command is `pnorm(-1.2)`.

   (d) $\mathrm{P}(-1.2 < Z < 1.2) = 2\mathrm{P}(0 < Z < 1.2) = 2(0.8849 - 0.5) = 0.7698$.
   R command is `pnorm(1.2)-pnorm(-1.2)`.

   (e) $\mathrm{P}(-0.81 < Z < 2.32) = \mathrm{P}(0 < Z < 0.81) + \mathrm{P}(0 < Z < 2.32) = (0.791 - 0.5) + (0.9898 - 0.5) = 0.7808$.
   R command is `pnorm(2.32)-pnorm(-0.81)`.

   (f) $\mathrm{P}(Z < a) = 0.67 \Rightarrow a = 0.44$.
   R command is `qnorm(0.67)`.

   (g) $\mathrm{P}(-b < Z < b) = 0.8 \Rightarrow \mathrm{P}(0 < Z < b) = 0.4 \Rightarrow \mathrm{P}(Z < b) = 0.4 + 0.5 = 0.9 \Rightarrow b = 1.28$.
   R command is `qnorm(0.9)`.

5. $X \sim \mathrm{N}(2, 3^2)$. Thus $Z = \frac{X-2}{3} \sim \mathrm{N}(0, 1)$.

6. $P(X > 8) = P\left(Z > \frac{8-2}{3}\right) = P(Z > 2.0) = 0.0228$.

7. $X \sim \mathrm{N}(10, 16)$. Thus $Z = \frac{X-10}{4} \sim \mathrm{N}(0, 1)$.

   (a) $\mathrm{P}(X > 12) = \mathrm{P}\left(Z > \frac{12-10}{4}\right) = \mathrm{P}(Z > 0.5) = 1 - \mathrm{P}(Z < 0.5) = 1 - 0.6915 = 0.3085$.
   R command is `pnorm(12,10,4,lower.tail = F)`.

   (b) $\mathrm{P}(X < k) = \mathrm{P}\left(Z < \frac{k-10}{4}\right) = 0.90$ and hence, $\frac{k-10}{4} = 1.28$.
   Thus $k = 10 + 1.28 \times 4 = 15.12$. R command is `qnorm(0.9,10,4)`.

Check with R

For binomial caculations: Use `dbinom(x,n,p)` and `pbinom(x,n,p)` for $\mathrm{P}(X = x)$ and $\mathrm{P}(X \leq x)$ respectively if $X \sim \mathrm{B}(\texttt{n},\texttt{p})$.

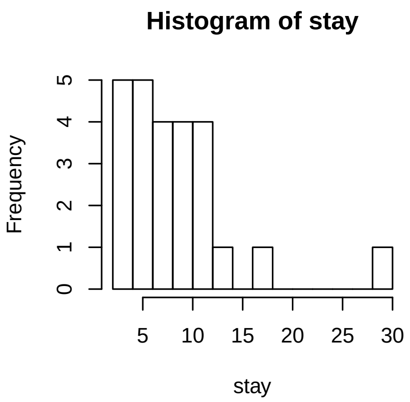
For normal caculations: Use `pnorm(x,m,s)` for $P(X < x)$ and `qnorm(p,m,s)` for a such that $P(X < a) = p$ if $X \sim N(m, s^2)$.

8. Let X be the number of hours per week spent watching TV.
   Therefore, $X \sim N(15, 2^2)$.
   Then $P(15 < X < 19) = 0.4772$. R command is `pnorm(19,15,2)-pnorm(15,15,2)`.

   Similarly, $P(X \geq 16) = 0.3085$ and $P(X \leq 14) = 0.3085$. R commands are `1-pnorm(16,15,2)` and `pnorm(14,15,2)` respectively.

9.
```
> dat = read.table(file="hospital.txt",skip=1)
> dat
   V1 V2 V3 V4   V5 V6 V7 V8 V9
1   1  5 30  2 99.0  8  2  2  1
2   2 10 73  2 98.0  5  2  1  1
3   3  6 40  2 99.0 12  2  2  2
4   4 11 47  2 98.2  4  2  2  2
5   5  5 25  2 98.5 11  2  2  2
6   6 14 82  1 96.8  6  1  2  2
7   7 30 60  1 99.5  8  1  1  1
8   8 11 56  2 98.6  7  2  2  1
9   9 17 43  2 98.0  7  2  2  1
10 10  3 50  1 98.0 12  2  1  2
11 11  9 59  2 97.6  7  2  1  1
12 12  3  4  1 97.8  3  2  2  2
13 13  8 22  2 99.5 11  1  2  2
14 14  8 33  2 98.4 14  1  1  2
15 15  5 20  2 98.4 11  2  1  2
16 16  5 32  1 99.0  9  2  2  2
17 17  7 36  1 99.2  6  1  2  2
18 18  4 69  1 98.0  6  2  2  2
19 19  3 47  1 97.0  5  1  2  1
20 20  7 22  1 98.2  6  2  2  2
21 21  9 11  1 98.2 10  2  2  2
22 22 11 19  1 98.6 14  1  2  2
23 23 11 67  2 97.6  4  2  2  1
24 24  9 43  2 98.6  5  2  2  2
25 25  4 41  2 98.0  5  2  2  1
> stay=dat[,2]
> age=dat[,3]
> hist(stay,breaks=10)  #9(a)
> hist(age,breaks=10)  #9(a)
> mu=mean(age)
> sd=sd(age)
> c(mu,sd)  #9(b)
[1] 41.24000 20.10240
> 1-pnorm(60,mu,sd)  #9(c)
[1] 0.1753528
```

(a) The distribution of age can be better approximated by normal than the distribution of stay because the distribution of age is more symmetric.



(b) The sample mean of age = 41.42 and sample s.d. = 20.1024.
(c) We have age $\sim N(41.42, 20.1024^2)$ approximately. P(age$> 60) = 0.1754$.

MATH 1015 - BIOSTATISTICS (Problem Sheet: Week 6, 2012 - Solutions)

(1)

(a) (i)
$$\begin{aligned} P(-1.72 < Z < 0.52) &= P(-1.72 < Z < 0) + P(0 < Z < 0.52) \\ &= (0.9573 - 0.5) + (0.9357 - 0.5) = 0.6558 \end{aligned}$$

(ii) $P(|Z| > 1.96) = P(Z < -1.96) + P(Z > 1.96) = 2P(Z > 1.96) = 2(1-0.975) = 0.05$

(b)
$$\begin{aligned} \text{(i) } a &= 0.61 \quad \text{since} \quad P(Z \leq 0.61) = 0.7291 \\ \text{(ii) } b &= 1.28 \quad \text{since} \quad P(Z > 1.28) = 0.10 \Rightarrow P(Z < 1.28) = 0.90 \\ \text{(iii) } c &= 1.645 \quad \text{since} \quad P(|Z| < c) = 2P(0 < Z < c) = 0.90 \\ &\Rightarrow P(0 < Z < c) = 0.45 \Rightarrow P(Z < c) = 0.5 + 0.45 = 0.95 \text{ and } P(Z < 1.645) = 0.95 \end{aligned}$$

(2) $X \sim N(100, 16)$. The prob. that the coil yields \$25 profit is

$$P(X > 95) = P\left(Z > \frac{95-100}{4}\right) = P(Z > -1.25) = P(Z < 1.25) = 0.8943.$$

and the prob. that the coil yields \$10 is $1 - 0.8943 = 0.1057$.

$$\text{Average profit} = \$25 \times 0.8943 + 10 \times 0.1057 = \$23.41.$$

(3) Let X denote intraocular pressure, $X \sim N(16, 4^2)$.

(a)
$$\begin{aligned} P(12 \leq X \leq 20) &= P\left(\frac{12-16}{4} \leq Z \leq \frac{20-16}{4}\right) = P(-1 \leq Z \leq 1) \\ &= 2P(0 \leq Z \leq 1) = 2(0.8413 - 0.5) = 0.6826 \end{aligned}$$

i.e. approx. 68.26% of unaffected adults fall in this range.

(b) $P(X > c) = 0.01 \Rightarrow P(X < c) = 0.99 \Rightarrow \dfrac{c-16}{4} = 2.33 \Rightarrow c = 25.32$
i.e. Pressures above 25.32 are considered abnormally high (units: 25.32mm Hg).

(4) R command are

(1)(a) `pnorm(0.52)-pnorm(-1.72)`; `2*(1-pnorm(1.96))`; (b) `qnorm(0.7291)`; `qnorm(1-0.1)`; `qnorm(0.5+0.9/2)`

(2) `25*(1-pnorm(95,100,4))+10*pnorm(95,100,4)`

(3) `pnorm(20,16,4)-pnorm(12,16,4)`; `qnorm(1-0.01,16,4)`