Semester 1	Solution to Tutorial Set 7	2013

1. (d) Since $n = 100$ and $p = 0.02$, the mean and variance of X are respectively

   $E(X) = \mu = np = 50 \times 0.02 = 1$ and $Var(X) = \sigma^2 = np(1-p) = 50 \times 0.02 \times 0.98 = 0.98$.

   Then the sampling distribution of $\bar{X}$ has mean μ and variance $\frac{\sigma}{n}$. Since the sample size $n = 100 > 30$, the distribution of $\bar{X}$ is approximately normal $N(\mu, \frac{\sigma^2}{n})$, i.e. $N(1, \frac{0.98}{100}) = N(1, 0.0098)$.

2. (d) Use $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ where $\mu = 100$, $\sigma^2 = 25$, and $n = 16$, i.e. $\bar{X} \sim N(100, \frac{25}{16})$

3. (e) Draw diagram to illustrate the area. $P(\overline{X} \leq 102) = P\left(Z \leq \frac{102-100}{5/4}\right) = P(Z \leq \frac{8}{5})$.

4. $P(Z \leq 1.6) = 0.9452$. R command is `pnorm(1.6)` (default mean is 0 and sd is 1).

5. We have $\mu = 200$, $\sigma = 5$ and $n = 16$. Draw diagrams to illustrate the areas.

   $P(X > 208) = P(Z > \frac{208-200}{5}) = P(Z > 1.6) = 1 - 0.9452 = 0.0548$.
   Since $\bar{X} \sim N(200, \frac{5^2}{16})$, $P(\bar{X} > 208) = P(Z > \frac{208-200}{5/4}) = P(Z > 6.4) = 0.0000$.

   It is very unlikely to observe the average of 208 calories or more in a sample of size 16.

6. (a) Standard normal (b) t distributed with $n-1$ df.

7. se$=\frac{s}{\sqrt{n}} = \frac{\sqrt{0.00639}}{\sqrt{27}} = \frac{0.080}{\sqrt{27}} = 0.015$.

8. From the t table with df=15 and upper percentile=$(1-0.9)/2 = 0.05$, $k = 1.753$. Draw diagram to illustrate the area.

9. (a) Use: $(\bar{x} - t_{n-1,\alpha/2} \times s/\sqrt{n},\ \bar{x} + t_{n-1,\alpha/2} \times s/\sqrt{n})$, where $t_{15,0.05} = 1.753$ from Q8,

   $n = 16$, $\bar{x} = 194.8$ and $s = 13.14$. Therefore, the 90% CI for μ is

   $$(194.8 - 1.753 \times 13.14/\sqrt{16},\ 194.8 + 1.753 \times 13.14/\sqrt{16} = (189.0414, 200.5586).$$

   (b) Now the upper percentile is $(1-0.95)/2 = 0.025$. From the t table, $t_{15,0.025} = 2.131$.

   Therefore, the 95% CI for μ is

   $$(194.8 - 2.131 \times 13.14/\sqrt{16},\ 194.8 + 2.131 \times 13.14/\sqrt{16}) = (187.7997, 201.8003).$$

10. Q5 Use `1-pnorm(208, 200, 5)`. Answer is 0.05479929.
    Use `1-pnorm(208, 200, 5/4)`. Answer is 7.768852×10^{-11}.
    Q8 The lower percentile is $1 - (1 - 0.9)/2 = 0.95$.
    Use `qt(0.95,15)`. Answer is 1.753050.

11. The R commands are

```
> x = scan(file=url("http://www.maths.usyd.edu.au/math1015/r/germination.txt"))
Read 65 items
> x
 [1] 84 88 81 91 84 88 85 90 88 85 87 93 90 88 85 87 90 85 85 89 89 91 85 79 90
[26] 90 86 89 92 80 88 84 89 90 86 92 87 89 85 84 82 89 89 89 85 86 86 88 87 83
[51] 89 87 87 91 86 91 93 94 88 88 93 87 87 91 89
> mean(x)
[1] 87.58462
> sd(x)
[1] 3.136939
> t.test(x)

        One Sample t-test

data:  x
t = 225.1015, df = 64, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 86.80732 88.36191
sample estimates:
mean of x
 87.58462
```

mean=87.585, sd=3.137 and the 95% CI =(86.807, 88.362)

Problem Sheet Week 7 - Solutions

1. Let $\overline{X}$ be the mean of a random sample of size 100. $\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$.
   Use $\mu = 69$ and $\frac{\sigma^2}{n} = \frac{48^2}{100} = 4.8^2$.
   (a) $P(\overline{X} > 80) = P(Z > \frac{80-69}{4.8}) = P(Z > 2.29) = 0.0110$. Use `1-pnorm(80, 69, 4.8)`.
   (b) $P(59 \le \overline{X} \le 79) = P(\frac{59-69}{4.8} < Z < \frac{79-69}{4.8}) = P(-2.08 \le Z \le 2.08) = 0.9628$.
   Use `pnorm(79, 69, 4.8)-pnorm(59, 69, 4.8)`.

2. $\mu = 5000$, $\sigma = 500$ and $n = 100$. $\bar{X} \sim N(5000, \frac{500^2}{100})$.
   (a) $P(\overline{X} < 4928) = P(Z < \frac{4928-5000}{50}) = P(Z < -1.44) = 0.0749$. Use `pnorm(4928, 5000, 50)`.
   (b) $P(4950 < \overline{X} < 5050) = P(\frac{4950-5000}{50} < Z < \frac{5050-5000}{50}) = P(-1 < Z < 1) = 0.68$.
   Use `pnorm(5050, 5000, 50)-pnorm(4950, 5000, 50)`.

3. $\bar{X} \sim N(70, \frac{10^2}{16})$. $P(\bar{X} > 75) = P(Z \ge \frac{75-70}{10/\sqrt{16}}) = P(Z \ge 2) = 1 - 0.9772 = 0.0228$. Use `1-pnorm(75, 70, 2.5)`.

4. 2.583, -1.313, 2.365. Use table 3 or R codes: `qt(0.99,16)`, `qt(0.1,28)`, `qt(0.975,7)`.

5. Use $(\bar{x} - t_{n-1,\alpha/2} \times s/\sqrt{n},\ \bar{x} + t_{n-1,\alpha/2} \times s/\sqrt{n})$, where $t_{101,0.975} = 1.984$ and $t_{68,0.975} = 1.995$. The calculation can be checked using R where the 8 CIs are calculated using vectors of 8 means, 8 sd and 8 n. The R codes are given below.

```
m=c(0.97,4.43,1.68,36.5,1,4.49,1.57,23.57)
s=c(0.22,0.64,0.47,16.08,0.19,0.71,0.4,13.79)
n=c(102,102,102,102,69,69,69,69)
lower=m-qt(0.975,n-1)*s/sqrt(n)
upper=m+qt(0.975,n-1)*s/sqrt(n)
CI=cbind(lower,upper)
CI
```

The results are

```
> CI
          lower     upper
[1,]  0.9267879  1.013212
[2,]  4.3042921  4.555708
[3,]  1.5876832  1.772317
[4,] 33.3415883 39.658412
[5,]  0.9543570  1.045643
[6,]  4.3194394  4.660561
[7,]  1.4739095  1.666090
[8,] 20.2572803 26.882720
```