TUTORIAL EXERCISE PACKAGE - 2013

MATH1015 - BIOSTATISTICS

Semester 1	Solution to Tutorial Set 8	2013

1. Answer: (b)

   Explanation: If everything else is kept the same, then smaller margin of error means smaller t or z, which means lower confidence level.

2. Answer: (a) The 95% CI for p is $\hat{p} \pm z \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.48 \pm 1.96 \times \sqrt{\frac{0.48 \times 0.52}{1000}} = 0.48 \pm 0.03097 =$ $(0.4490, 0.5110)$

3. (a) Since $p = 0.5$ lies inside 95% CI, (0.4490,0.5110), the data are consistent with $H_0 : p = 0.5$.

   (b) Since $p = 0.4$ lies outside 95% CI, (0.4490,0.5110), the data are against $H_0 : p = 0.4$.

4. Answer (c) The observed test statistic $= \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{7-6}{2/\sqrt{16}} = 2$

5. Since the sample size is 16, df=15 and large observed values of $\bar{x}$ argue against H_0, the P-value $= \text{P}(t_{15} \geq 2.00)$. Using the t-table, the P-value is between 0.025 and 0.05.

6. Answer (a). Reason: the P-value is less than 0.05.

7. $n = 22$, sample total=191.4 and $s = 2.25$. The point estimate of μ is $\bar{x} = \frac{191.4}{22} = 8.7$.

   Use the one sample t-test to test $H_0 : \mu = 10$ against $H_1 : \mu < 10$.

   $$t_{obs} = \frac{8.7 - 10}{2.25/\sqrt{22}} = -2.71.$$

   Small values of t_{obs} argue against H_0 and therefore, the P-value is

   $$P\text{-value} = P(t_{21} < -2.71) < 0.01$$

   Since the P-value is *very small*, we have *very strong evidence* against H_0.

8. Test $H_0 : \mu = 300$ against $H_1 : \mu \neq 300$.

   Use the one-sample two-sided t-test to test this claim.

   $$t_{obs} = \frac{295 - 300}{20/\sqrt{46}} = -1.696.$$

   Either small or large values of t_{obs} argue against H_0 and therefore, the P-value is

   $$P\text{-value} = 2P(t_{45} \leq -1.696) \in (2 \times 0.025, 2 \times 0.05) = (0.05, 0.1)$$

   Clearly, the P-value is *above* 0.05 and the data are consistent with H_0.

   NOTE: In this case, the sample size is large and the t-test is almost identical to the test using normal instead of t distribution.

9. The R commands are
   `1-pt((7-6)/(2/sqrt(16)),15)`,
   `pt((191.4/22-10)/(2.25/sqrt(22)),21)` and
   `2*pt((295-300)/(20/sqrt(46)),45)` respectively.
   The answers are
   `0.03197250`,
   `0.006557571` and
   `0.0968744` respectively.

10.
```
> t.test(x, mu=86)


        One Sample t-test


data:  x
t = 4.0726, df = 64, p-value = 0.0001304
alternative hypothesis: true mean is not equal to 86
95 percent confidence interval:
 86.80732 88.36191
sample estimates:
mean of x
 87.58462
```
   Since the p-value $= 0.0001304 < 0.05$, the data are against H_0.

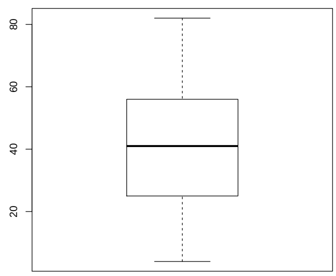
11.
```
> dat=read.table(file=url("http://www.maths.usyd.edu.au/math1015/r/hospital.txt"),skip=1)
> dat
   V1 V2 V3 V4   V5 V6 V7 V8 V9
1   1  5 30  2 99.0  8  2  2  1
2   2 10 73  2 98.0  5  2  1  1
3   3  6 40  2 99.0 12  2  2  2
4   4 11 47  2 98.2  4  2  2  2
5   5  5 25  2 98.5 11  2  2  2
6   6 14 82  1 96.8  6  1  2  2
7   7 30 60  1 99.5  8  1  1  1
8   8 11 56  2 98.6  7  2  2  1
9   9 17 43  2 98.0  7  2  2  1
10 10  3 50  1 98.0 12  2  1  2
11 11  9 59  2 97.6  7  2  1  1
12 12  3  4  1 97.8  3  2  2  2
13 13  8 22  2 99.5 11  1  2  2
14 14  8 33  2 98.4 14  1  1  2
```

```
15 15  5 20  2 98.4 11  2  1  2
16 16  5 32  1 99.0  9  2  2  2
17 17  7 36  1 99.2  6  1  2  2
18 18  4 69  1 98.0  6  2  2  2
19 19  3 47  1 97.0  5  1  2  1
20 20  7 22  1 98.2  6  2  2  2
21 21  9 11  1 98.2 10  2  2  2
22 22 11 19  1 98.6 14  1  2  2
23 23 11 67  2 97.6  4  2  2  1
24 24  9 43  2 98.6  5  2  2  2
25 25  4 41  2 98.0  5  2  2  1

> boxplot(age)
> length(age)
[1] 25
> t.test(age,mu=40,alt='greater')

        One Sample t-test

data:  age
t = 0.3084, df = 24, p-value = 0.3802
alternative hypothesis: true mean is greater than 40
95 percent confidence interval:
 34.36143      Inf
sample estimates:
mean of x
    41.24
```



Since the sample size is 25 (small), t-test with normality assumption should be used. Since the p-value $= 0.3802 > 0.05$, the data are against H_0.

Note that `t.test` provides the default 95% CI for μ. If we need a different confidence level, say 94%, then we can use `t.test(x, mu = 11, conf.level = 0.94)`

By default `t.test` tests two-sided alternative hypothesis. We can change it to lower or uppersided alternative, by using the options `alt = 'less'` (or `alternate='l'`) and `alt = 'greater'`, respectively. For example, `t.test(x, mu = 11, alt = 'greater')` tests $H_0 : \mu = 11$ vs. $H_1 : \mu > 11$.

Answers to Some of the Additional Problems

1.(Q7.26)
Test $H_0 : \mu = 230$ against $H_1 : \mu \neq 230$.
$t_{obs} = -7.70$. df=23
P-value is less than 0.001.
We have strong evidence against H0.
2.(Q7.27)
A 95% CI for μ is $175 \pm 2.069 \times 35/\sqrt{24}$ or (160.2, 189.8).
3. (Q7.28)
This 95% CI in (2) does not contain and and is significantly lower than 230.
The hypothesis test tells us precisely how significant the results are. The CI gives a range of values within which the true mean is likely to fall.