Table 1: Estimators and variance estimates from a SRS (Ch.1 & 3)

Par.	**Ordinary**	**Ratio**	**Regression**	**Hartley-Ross**
Ratio R	$\frac{\bar{y}}{\bar{X}}$ $\frac{1}{\bar{X}^2}(1-\frac{n}{N})\frac{s_y^2}{n}$	$\frac{\bar{y}}{\bar{x}}$ $\frac{1}{\bar{X}^2}(1-\frac{n}{N})\frac{s_r^2}{n}$	- -	$\bar{r}' + \frac{N-1}{N\bar{X}}\ \frac{n(\bar{y}-\bar{r}'\bar{x})}{n-1}$ $\bar{r}' = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{x_i}$
Mean $\bar{Y}$	$\bar{y}$ $(1-\frac{n}{N})\frac{s_y^2}{n}$	$\frac{\bar{y}}{\bar{x}}\bar{X}$ $(1-\frac{n}{N})\frac{s_r^2}{n}$ $\mathrm{var}(\hat{\bar{Y}}_r) < \mathrm{var}(\hat{\bar{Y}})$ if $\hat{\rho} > \frac{\bar{y}s_x}{2\bar{x}s_y}$	$\overline{y} + \frac{s_{xy}}{s_x^2}(\overline{X}-\overline{x})$ $(1-\frac{n}{N})\frac{s_y^2(1-\hat{\rho}^2)}{n}$ $\mathrm{var}(\hat{\bar{Y}}_{reg}) < \mathrm{var}(\hat{\bar{Y}}_r)$ equal if $b = r = \frac{\bar{y}}{\bar{x}}$	$\bar{X}\bar{r}' + \frac{N-1}{N}\frac{n(\bar{y}-\bar{r}'\bar{x})}{n-1}$ -
Total Y	$N\bar{y}$ $N^2(1-\frac{n}{N})\frac{s_y^2}{n}$	$\frac{\bar{y}}{\bar{x}}X$ $N^2(1-\frac{n}{N})\frac{s_r^2}{n}$	$N\overline{y} + \frac{s_{xy}}{s_x^2}(X-N\overline{x})$ $N^2(1-\frac{n}{N})\frac{s_y^2(1-\hat{\rho}^2)}{n}$	$X\bar{r}' + (N-1)\frac{n(\bar{y}-\bar{r}'\bar{x})}{n-1}$ -

1. $s_y^2 = \frac{1}{n-1}\Big(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2\Big)$,
   $s_r^2 = \frac{1}{n-1}\Big(\sum_{i=1}^{n} y_i^2 - 2r\sum_{i=1}^{n} x_i y_i + r^2\sum_{i=1}^{n} x_i^2\Big) = s_y^2 - 2r\hat{\rho}s_x s_y + r^2 s_x^2,\ r = \frac{\bar{y}}{\bar{x}}$,
2. For count data, replace $\bar{y}$ by sample proportion p and s_y^2 by $\frac{n}{n-1}p(1-p)$ or $p(1-p)$ approx.

Table 2: Estimators and variance estimates for a poststratified SRS (Ch.1)

Par.	Estimator	N_l	Variance	Remarks
$\bar{Y}_l$	$\hat{\bar{Y}}_{pst,lm1} = \dfrac{\bar{y}'}{W_l}$	X	$\text{var}(\hat{\bar{Y}}_{pst,lm1}) = \dfrac{1}{W_l^2}\left(1-\dfrac{n}{N}\right)\dfrac{s_l'^2}{n}$	$y_i' = y_i$ if $i \in C_l$ & 0 otherwise
Y_l	$\hat{Y}_{pst,lm1} = N\bar{y}'$	$\surd$	$\text{var}(\hat{Y}_{pst,lm1}) = N^2\left(1-\dfrac{n}{N}\right)\dfrac{s_l'^2}{n}$	$\bar{y}' = \dfrac{\sum_{i=1}^n y_i'}{n}$, $s_l'^2 = \dfrac{\sum_{i=1}^n y_i'^2 - n\bar{y}'^2}{n-1}$
$\bar{Y}_l$	$\hat{\bar{Y}}_{pst,lm2} = \dfrac{n}{n_l}\bar{y}' = \bar{y}$	$\surd$	$\text{var}(\hat{\bar{Y}}_{pst,lm2}) \simeq \dfrac{1}{W_l}\left(1-\dfrac{n}{N}\right)\dfrac{s_l^2}{n}$	$\bar{y} = \dfrac{1}{n_l}\sum_{i\in C_l} y_i$
Y_l	$\hat{Y}_{pst,lm2} = N_l\bar{y}$	X	$\text{var}(\hat{Y}_{pst,lm2}) \simeq N^2 W_l\left(1-\dfrac{n}{N}\right)\dfrac{s_l^2}{n}$	$s_l^2 = \dfrac{1}{n_l-1}\sum_{i\in C_l} y_i{}^2 - n_l\bar{y}^2$
$\overline{Y}$	$\hat{\bar{Y}}_{pst} = \sum_{l=1}^L W_l\bar{y}_l$	X	$\text{var}(\hat{\bar{Y}}_{pst}) \simeq \left(1-\dfrac{n}{N}\right)\sum_{l=1}^L W_l\dfrac{s_l^2}{n}$	Extra $=\dfrac{1}{n^2}\sum_{l=1}^L(1-W_l)s_l^2$
Y	$\hat{Y}_{pst} = \sum_{l=1}^L N_l\bar{y}_l$	X	$\text{var}(\hat{Y}_{pst}) \simeq N^2\left(1-\dfrac{n}{N}\right)\sum_{l=1}^L W_l\dfrac{s_l^2}{n}$	Extra $=\dfrac{N^2}{n^2}\sum_{l=1}^L(1-W_l)s_l^2$

1. 'X' indicates that the estimate cannot be applied if N_l and hence W_l is unknown.
2. $E\left(\dfrac{1}{n_l}\right) = \dfrac{1}{nW_l} + \dfrac{1-W_l}{n^2W_l^2} \simeq \dfrac{1}{nW_l}$. The extra var. $\dfrac{1-W_l}{n^2W_l^2}$ is ignored in the var. formulae.

Table 3: Sample size determination for SRS and stratified SRS (Ch.1 & 2)

SRS for mean $\hat{\bar{Y}}$ s.t. $\Pr(\lvert\bar{y}-\overline{Y}\rvert \le \delta_\mu) = 0.95$ and $S^2 = p(1-p)$ for proportion $\hat{P}$	
Sample n	$n \ge \dfrac{NS^2}{N(\delta_\mu)^2/z_{\alpha/2}^2 + S^2} \overset{f=0}{\approx} \dfrac{z_{\alpha/2}^2 S^2}{(\delta_\mu)^2}$

Stratified SRS for mean $\hat{\bar{Y}}$ s.t. $\Pr(\lvert\bar{y}_{st}-\overline{Y}\rvert \le \delta_\mu) = 0.95$ and $V = \dfrac{(\delta_\mu)^2}{z_{\alpha/2}^2}$			
Sizes	**Optimum**	**Neyman**	**Proportional**
Stratum n_l	$n_l = n\left(\dfrac{\frac{W_lS_l}{\sqrt{C_l}}}{\sum_{i=1}^L \frac{W_iS_i}{\sqrt{C_i}}}\right)$	$n_l = n\left(\dfrac{W_lS_l}{\sum_{i=1}^L W_iS_i}\right)$	$n_l = nW_l$
Sample n	$n \ge \dfrac{\left(\sum_{l=1}^L W_lS_l\sqrt{C_l}\right)\left(\sum_{i=1}^L W_iS_i/\sqrt{C_i}\right)}{V + \frac{1}{N}\sum_{l=1}^L W_lS_l^2}$	$n \ge \dfrac{\left(\sum_{l=1}^L W_lS_l\right)^2}{V + \frac{1}{N}\sum_{l=1}^L W_lS_l^2}$	$n \ge \dfrac{\sum_{l=1}^L W_lS_l^2}{V + \frac{1}{N}\sum_{l=1}^L W_lS_l^2}$

Table 4: Estimators and variance estimates for stratified SRS (Ch.2 & Ch.3)

Parameter	Estimator	Variance
Ordinary estimator $s_{yl}^2 = \frac{1}{n_l-1}\left(\sum_{i=1}^{n_l} y_{li}^2 - n_l\bar{y}_l^2\right), \quad W_l = \frac{N_l}{N}$		
Ratio $R = \frac{1}{\bar{X}}\sum_{l=1}^{L} W_l\bar{Y}_l$	$\widehat{R}_{st} = \frac{1}{\bar{X}}\sum_{l=1}^{L} W_l\bar{y}_l$	$\text{var}(\widehat{R}_{st}) = \frac{1}{\bar{X}^2}\sum_{l=1}^{L} W_l^2\left(1-\frac{n_l}{N_l}\right)\frac{s_{yl}^2}{n_l}$
Mean $\bar{Y} = \sum_{l=1}^{L} W_l\bar{Y}_l$	$\widehat{\bar{Y}}_{st} = \sum_{l=1}^{L} W_l\bar{y}_l$	$\text{var}(\widehat{\bar{Y}}_{st}) = \sum_{l=1}^{L} W_l^2\left(1-\frac{n_l}{N_l}\right)\frac{s_{yl}^2}{n_l}$
Total $Y = N\sum_{l=1}^{L} W_l\bar{Y}_l$	$\widehat{Y}_{st} = N\sum_{l=1}^{L} W_l\bar{y}_l$	$\text{var}(\widehat{Y}_{st}) = N^2\sum_{l=1}^{L} W_l^2\left(1-\frac{n_l}{N_l}\right)\frac{s_{yl}^2}{n_l}$
Separate ratio estimator $s_{sr,l}^2 = s_{yl}^2 - 2\,r_l\hat{\rho}s_{xl}s_{yl} + r_l^2 s_{xl}^2, \quad r_l = \frac{\bar{y}_l}{\bar{x}_l}$		
Ratio $R = \frac{1}{\bar{X}}\sum_{l=1}^{L} W_l\bar{X}_l R_l$	$\widehat{R}_{st,sr} = \frac{1}{\bar{X}}\sum_{l=1}^{L} W_l\bar{X}_l r_l$	$\text{var}(\widehat{R}_{st,sr}) = \frac{1}{\bar{X}^2}\sum_{l=1}^{L} W_l^2\left(1-\frac{n_l}{N_l}\right)\frac{s_{sr,l}^2}{n_l}$
Mean $\bar{Y} = \sum_{l=1}^{L} W_l\bar{X}_l R_l$	$\widehat{\bar{Y}}_{st,sr} = \sum_{l=1}^{L} W_l\bar{X}_l r_l$	$\text{var}(\widehat{\bar{Y}}_{st,sr}) = \sum_{l=1}^{L} W_l^2\left(1-\frac{n_l}{N_l}\right)\frac{s_{sr,l}^2}{n_l}$
Total $Y = N\sum_{l=1}^{L} W_l\bar{X}_l R_l$	$\widehat{Y}_{st,sr} = N\sum_{l=1}^{L} W_l\bar{X}_l r_l$	$\text{var}(\bar{Y}_{st,sr}) = N^2\sum_{l=1}^{L} W_l^2\left(1-\frac{n_l}{N_l}\right)\frac{s_{sr,l}^2}{n_l}$
Combine ratio estimator $s_{cr,l}^2 = s_{yl}^2 - 2\,r_c\hat{\rho}s_{xl}s_{yl} + r_c^2 s_{xl}^2, \quad r_{st,cr} = \frac{\sum_{l=1}^{L} W_l\bar{y}_l}{\sum_{l=1}^{L} W_l\bar{x}_l}$		
Ratio $R_{st,cr} = \frac{\sum_{l=1}^{L} W_l\bar{Y}_l}{\sum_{l=1}^{L} W_l\bar{X}_l}$	$\widehat{R}_{st,cr} = \frac{\sum_{l=1}^{L} W_l\bar{y}_l}{\sum_{l=1}^{L} W_l\bar{x}_l} = r_{st,cr}$	$\text{var}(\widehat{R}_{st,cr}) = \frac{1}{\bar{X}^2}\sum_{l=1}^{L} W_l^2\left(1-\frac{n_l}{N_l}\right)\frac{s_{cr,l}^2}{n_l}$
Mean $\bar{Y} = \bar{X}R_{st,cr}$	$\widehat{\bar{Y}}_{st,cr} = \bar{X}r_{st,cr}$	$\text{var}(\widehat{\bar{Y}}_{st,cr}) = \sum_{l=1}^{L} W_l^2\left(1-\frac{n_l}{N_l}\right)\frac{s_{cr,l}^2}{n_l}$
Total $Y = N\bar{X}R_{st,cr}$	$\widehat{Y}_{st,cr} = N\bar{X}r_{st,cr}$	$\text{var}(\bar{Y}_{st,cr}) = N^2\sum_{l=1}^{L} W_l^2\left(1-\frac{n_l}{N_l}\right)\frac{s_{cr,l}^2}{n_l}$
Double sampling estimator		
Mean $\overline{Y}_{st} = \sum_{l=1}^{L} W_l\overline{Y}_l$	$\widehat{\overline{Y}}_{st,ds} = \sum_{l=1}^{L} w_l\bar{y}_l$ $w_l = \frac{n'_l}{n'}$	$\text{var}(\widehat{\overline{Y}}_{st,ds}) = \sum_{l=1}^{L}\left[\frac{w_l^2 s_l^2}{n_l} + \frac{w_l(\bar{y}_l - \widehat{\overline{Y}}_{st,ds})^2}{n'}\right]$ If n' & N are large and $\frac{n_l}{N_l}$ are all small

Table 5: Systematic Sampling (Ch.4)

Parameter	Point Estimate	Variance
One systematic sample		
Mean $\overline{Y}$	$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$	$\text{Var}(\hat{\overline{Y}}_{sy}) = \frac{1}{k}\sum_{j=1}^{k}(\overline{y}_j - \overline{y})^2$
(Sampling interval $k = \frac{N}{n}$)		$= \frac{N-1}{N}S^2 - \frac{N-k}{N}S_w^2$
		where $S_w^2 = \frac{1}{N-k}\left[\sum_{i=1}^{n}\sum_{j=1}^{k}(y_{ij} - \bar{y}_i)^2\right]$
		$S^2 = \frac{1}{N-1}\left[\sum_{i=1}^{n}\sum_{j=1}^{k}(y_{ij} - \overline{y})^2\right]$
		$\text{Var}(\hat{\overline{Y}}_{sy}) < \text{Var}(\hat{\overline{Y}}_{srs})$ if $S_w^2 > S^2$
Repeated systematic samples		
Mean $\overline{Y}$	$\overline{\overline{y}} = \frac{1}{n_s}\sum_{j=1}^{n_s}\overline{y}_j$	$\text{var}(\hat{\overline{Y}}_{sy,rs}) = \left(1 - \frac{n}{N}\right)\frac{s_{\overline{y}}^2}{n_s}$
(Sampling interval $k' = \frac{N}{n}\, n_s$)		$s_{\overline{y}}^2 = \frac{1}{n_s - 1}\sum_{j=1}^{n_s}(\overline{y}_j - \overline{\overline{y}})^2$
		$= \frac{1}{n_s - 1}\left(\sum_{j=1}^{n_s}\overline{y}_j^2 - n_s\overline{\overline{y}}^2\right)$

Table 6: Cluster Sampling (Ch.4)

Parameter	Ord. est.	Ratio est.	Var. in stage 1	Additional var. in stage 2
	$\bar{y}$	$r = \bar{y}/\bar{m}$		$\hat{y}_i = M_i\bar{y}_{i\cdot} = \frac{M_i}{m_i}\sum_{i=1}^{m_i} y_{ij}$ replace y_i
		$\bar{m} = \frac{1}{n}\sum_{i=1}^{n} M_i$		$\hat{r} = \bar{\hat{y}}/\bar{m}$ replace r
	s_y^2	s_r^2		$\bar{\hat{y}} = \frac{1}{n}\sum_{i=1}^{n}\hat{y}_i$ replace $\bar{y}$
Mean/ele. R	$\frac{\bar{y}}{\bar{M}}$ X	r $\checkmark$	$\frac{1}{\bar{M}^2}\left(1 - \frac{n}{N}\right)\frac{s^2}{n}$	$\frac{N}{nM^2}\sum_{i=1}^{n} M_i^2\left(1 - \frac{m_i}{M_i}\right)\frac{s_{yi}^2}{m_i}$
Mean/clus. $\bar{Y}$	$\bar{y}$ $\checkmark$	$\bar{M}r$ X	$\left(1 - \frac{n}{N}\right)\frac{s^2}{n}$	$\frac{1}{nN}\sum_{i=1}^{n} M_i^2\left(1 - \frac{m_i}{M_i}\right)\frac{s_{yi}^2}{m_i}$
Total Y	$N\bar{y}$ $\checkmark$	Mr X	$N^2\left(1 - \frac{n}{N}\right)\frac{s^2}{n}$	$\frac{N}{n}\sum_{i=1}^{n} M_i^2\left(1 - \frac{m_i}{M_i}\right)\frac{s_{yi}^2}{m_i}$

Note: 1. 'X' indicates that the estimate cannot be applied if $\bar{M} = M/N$ or M is unknown.

2. $\text{Var}(\hat{\overline{Y}}_{c1}) < \text{Var}(\hat{\overline{Y}}_{srs})$ if $M_i = \bar{M}$ and $S_b^2 < S^2$.

Table 7: Stratified one-stage cluster sample

Parameter	Estimate	Variance
Ordinary estimator: $\bar{y}_l = \frac{1}{n_l}\sum_{i=1}^{n_l} y_{li}$, $s_{yl}^2 = \frac{1}{n_l-1}\sum_{i=1}^{n_l}(y_{li}-\bar{y}_l)^2$, $\bar{M} = \frac{M}{N}$		
Mean/ele. R	$\widehat{R}_{stc1} = \frac{1}{\bar{M}}\sum_{l=1}^{L} W_l \bar{y}_l$ X	$\text{var}(\widehat{R}_{stc1}) = \frac{1}{\bar{M}^2}\sum_{l=1}^{L} W_l^2 \left(1-\frac{n_l}{N_l}\right)\frac{s_{yl}^2}{n_l}$
Mean/clus. $\bar{Y}$	$\widehat{\bar{Y}}_{stc1} = \sum_{l=1}^{L} W_l \bar{y}_l$ $\surd$	$\text{var}(\widehat{\bar{Y}}_{stc1}) = \sum_{l=1}^{L} W_l^2 \left(1-\frac{n_l}{N_l}\right)\frac{s_{yl}^2}{n_l}$
Total Y	$\widehat{Y}_{stc1} = \sum_{l=1}^{L} N_l\ \bar{y}_l$ $\surd$	$\text{var}(\widehat{Y}_{stc1}) = \sum_{l=1}^{L} N_l^2 \left(1-\frac{n_l}{N_l}\right)\frac{s_{yl}^2}{n_l}$
Separate ratio estimator: $r_l = \frac{\sum_{i=1}^{n_l} y_{li}}{\sum_{i=1}^{n_l} M_{li}}$, $s_{srl}^2 = s_{yl}^2 - 2r_l s_{xyl} + r_l^2 s_{xl}^2$, $\bar{M}_l = \frac{M_l}{N_l}$		
Mean/ele. R	$\widehat{R}_{stc1,sr} = \frac{1}{\bar{M}}\sum_{l=1}^{L} W_l\ r_l \bar{M}_l$ X	$\text{var}(\widehat{R}_{stc1,sr}) = \frac{1}{\bar{M}^2}\sum_{l=1}^{L} W_l^2 \left(1-\frac{n_l}{N_l}\right)\frac{s_{srl}^2}{n_l}$
Mean/clus. $\bar{Y}$	$\widehat{\bar{Y}}_{stc1,sr} = \sum_{l=1}^{L} W_l\ r_l \bar{M}_l$ X	$\text{var}(\widehat{\bar{Y}}_{stc1,sr}) = \sum_{l=1}^{L} W_l^2 \left(1-\frac{n_l}{N_l}\right)\frac{s_{srl}^2}{n_l}$
Total Y	$\widehat{Y}_{stc1,sr} = \sum_{l=1}^{L} N_l\ r_l \bar{M}_l$ X	$\text{var}(\widehat{Y}_{stc1,sr}) = \sum_{l=1}^{L} N_l^2 \left(1-\frac{n_l}{N_l}\right)\frac{s_{srl}^2}{n_l}$
Combine ratio estimator: $r_c = \frac{\sum_{l=1}^{L} W_l\ \bar{y}_l}{\sum_{l=1}^{L} W_l\ \bar{m}_l}$, $s_{crl}^2 = s_{yl}^2 - 2r_c s_{xyl} + r_c^2 s_{xl}^2$		
Mean/ele. R	$\widehat{R}_{stc1,cr} = r_c$ $\surd$	$\text{var}(\widehat{R}_{stc1,cr}) = \frac{1}{\bar{M}^2}\sum_{l=1}^{L} W_l^2 \left(1-\frac{n_l}{N_l}\right)\frac{s_{crl}^2}{n_l}$
Mean/clus. $\bar{Y}$	$\widehat{\bar{Y}}_{stc1,cr} = \bar{M}\ r_c$ X	$\text{var}(\widehat{\bar{Y}}_{stc1,cr}) = \sum_{l=1}^{L} W_l^2 \left(1-\frac{n_l}{N_l}\right)\frac{s_{crl}^2}{n_l}$
Total Y	$\widehat{Y}_{stc1,cr} = M\ r_c$ X	$\text{var}(\widehat{Y}_{stc1,cr}) = \sum_{l=1}^{L} N_l^2 \left(1-\frac{n_l}{N_l}\right)\frac{s_{crl}^2}{n_l}$

'X' indicates that the estimate cannot be applied if $\bar{M}$ or M is unknown.

Table 8: Sampling with unequal probabilities (Ch.5)

Parameter	Estimate	Variance
Hansen-Hurwitz estimator for WR sample with sel. prob. p_i (PPS: $p_i = \frac{x_i}{X}$) & $z_i = \frac{y_i}{p_i}$		
Mean $\overline{Y}$	$\hat{\overline{Y}}_{HH} = \frac{1}{Nn}\sum_{i=1}^{n}\frac{y_i}{p_i} = \frac{\bar{z}}{N}$	$\text{var}(\hat{\overline{Y}}_{HH}) = \frac{1}{N^2 n(n-1)}\left(\sum_{i=1}^{n} z_i^2 - n\bar{z}^2\right) = \frac{s_z^2}{N^2 n}$
Total Y	$\hat{Y}_{HH} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i} = \bar{z}$	$\text{var}(\hat{Y}_{HH}) = \frac{1}{n(n-1)}\left(\sum_{i=1}^{n} z_i^2 - n\bar{z}^2\right) = \frac{s_z^2}{n}$
Horvitz-Thompson estimator for any sampling with inclusion probabilities π_i ($\sum_{i=1}^{n}\pi_i = n$)		
Mean $\overline{Y}$	$\hat{\overline{Y}}_{HT} = \frac{1}{N}\sum_{i=1}^{n}\frac{y_i}{\pi_i}$	For any sampling design: $\text{var}(\hat{\overline{Y}}_{HT,1}) = \frac{1}{N^2}\left(\sum_{i=1}^{n}\frac{1-\pi_i}{\pi_i^2}\,y_i^2 + 2\sum_{i=1}^{n}\sum_{j<i}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}\,y_i y_j\right)$ For samping WOR: $\text{var}(\hat{\overline{Y}}_{HT,2}) = \frac{1}{N^2}\left[\sum_{i=1}^{n}\sum_{j<i}\frac{\pi_i\pi_j-\pi_{ij}}{\pi_{ij}}\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2\right]$ For systematic samping (Hartley & Rao (1962) estimator): $\text{var}(\hat{\overline{Y}}_{HR,ipps}) = \frac{1}{N^2}\left[\sum_{i=1}^{n}\left(1-\frac{n-1}{n}\,\pi_i\right)\left(\frac{y_i}{\pi_i}-\frac{\hat{Y}_{sys}}{n}\right)^2\right]$
Total Y	$\hat{Y}_{HT} = \sum_{i=1}^{n}\frac{y_i}{\pi_i}$	For any sampling design: $\text{var}(\hat{Y}_{HT,1}) = \sum_{i=1}^{n}\frac{1-\pi_i}{\pi_i^2}\,y_i^2 + \sum_{i=1}^{n}\sum_{j\neq i}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}\,y_i y_j$ For samping WOR: $\text{var}(\hat{Y}_{HT,2}) = \sum_{i,j\in\mathcal{S}}\sum_{i<j}\frac{\pi_i\pi_j-\pi_{ij}}{\pi_{ij}}\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2$ For systematic samping (Hartley & Rao (1962) estimator): $\text{var}(\hat{Y}_{HR,ipps}) = \sum_{i\in\mathcal{S}}\left(1-\frac{n-1}{n}\,\pi_i\right)\left(\frac{y_i}{\pi_i}-\frac{\hat{Y}_{sys}}{n}\right)^2$