

Errata and Addenda for “Algebraic Invariants of Links”

2 April 2009

New references and substantial changes are starred.

My thanks to Sergey Melikhov for his observations [June 2008].

Chapter 1:

*page 5 line -8: “weight” was not defined! The weight of a group G is the minimal number of elements in a subset S such that $\langle\langle S \rangle\rangle_G = G$.

page 5 line -1: “group group” should be “group”.

page 6 line -10: “conection” should be “connection”.

page 9 line 1: “2” should be “2”.

page 10 line 4: See Theorem 2.2 of [Bir] for this assertion.

page 13 line -10: “successively” is misspelt.

*page 17, lines 14-16: replace these two sentences by “The links L and $h(L)$ are then said to be obtained from each other by an *elementary surgery*”.

*page 18, line 5: “ $T_i(sz, s)$ ” should be “ $T_i(s^{\pm 1}z, s)$ ”. (The oversight here affected also Theorem 6.11 and 7.1).

*page 18, lines 22-25: No. the ribbon group associated to such a ribbon has a 2-generator, 1-relator presentation, and so the knot module of a ribbon knot bounding a ribbon of this form can be generated by 2 elements. Counterexample: $(3_1\# - 3_1)\#(3_1\# - 3_1)$. I suspect that 9_{41} may also be a counterexample, as it bounds a ribbon disc with $Z/3Z$ symmetry.

*page 21 line 8: replace the sentence beginning on this line with “Thus Example 12 of [Fo62] is not ribbon *[Yj64] (see also Theorems 4.3 and 4.6 below), although it is slice [Ke65].”

page 23 line -8: add “. See Figure 3(a) of [Wa94]” before the “”.

Chapter 2:

page 32 line 11: “ $I_q(u, \partial v)$ ” should be “ $I_{q+1}(u, \partial v)$ ”.

page 32 line -7: the subscript “ $g \in Z^\mu$ ” should be “ $g \in \pi/H$ ”.

*page 34 (proof of 2.1): the arguments here deserve more explanation. The Snake lemma shows that $t - 1$ is injective on T and that $T/(t - 1)T \leq Z^{\mu-1}$. Since T is a noetherian torsion Λ -module it follows that $T = (t - 1)T$. The observation that H_{n+1} is free should have been made before estimating its

rank. Since an onto endomorphism of a noetherian module (such as H_n) is an isomorphism, $Z \otimes H_{n+1} \cong Z^{\mu-1}$.

page 36, line 15: “ $\theta_p D_p$ ” should be “ $\theta_{2-p} D_{3-p}$ ”.

*page 36, lines -11 to -9: “link module sequence” is defined in Chapter 4.

*pages 37-38: J.C.Cha has pointed out that there may be a gap in the proof of Theorem 2.3. (He used it in his work on invariants of equivariant concordance *[Cha04].) I have yet to check/repair/concede this point.

page 38, line 18: “ X ” should be “ Z ”.

page 38, line -11: “ d ” should be “ ∂ ”.

page 39 lines 12, 13: “ C_n to ... $(-1)^{n+1}$ ” should be “ C_{2q+1} to ... $(-1)^q$ ”.

page 42 line -12: “ b_N ” should be “ $-b_N$ ”.

Chapter 3:

*page 47 line -5: “ $r \in M$ ” should be “ r some nonzero divisor of R ”.

*page 53 line 22: “Chapter 6” should be “Theorem 3.22 below”.

*page 57 lines 18-20: The definition of resultant in this sentence is not correct. However the next sentence is correct and the resultant is used correctly below. (See [Lan], page 135). If θ is monic and $M = R[u, u^{-1}]/(\theta, \psi)$, considered as an R -module then it may easily be shown that $E_0(M) = (Res_R(\theta, \psi))$.

*page 62 lines 2-9: “ r ” should be “ $q - r$ ” throughout this proof, except in the final sentence.

Chapter 4:

page 71 line 9: there is no need to invoke Hilbert’s Syzygy Theorem here, for if M is a $R\Lambda_\mu$ -module which is free as an R -module and $K(\Lambda_\mu)_$ is the Koszul complex given below then $M \otimes_{\mathbb{Z}} K(\Lambda_\mu)_*$ with the diagonal Z^μ -action is a free resolution of M of length μ . In general, if $\phi : F \rightarrow M$ is an epimorphism from a free $R\Lambda_\mu$ -module F then splicing such a resolution of $\text{Ker}(\phi)$ with ϕ gives a free resolution of M of length $\mu + 1$.

page 74, lines 15-20: when $\mu = 1$ (the knot-theoretic case) I_μ is free. In this case we may obtain a square presentation matrix for B by adding a relation to kill the generator of A/B , without using “projectives are free”.

*page 80 line 12: we should assume here that $\alpha(L) = \mu$.

page 82 line 13: “ O ” should be “ 0 ”.

page 91 line 7: “ O ” should be “ 0 ”.

*page 92 lines 5-8: this deserves more explanation. The corresponding result for field coefficients follows easily from determinantal characterizations of the rank. In the integral case one reduces firstly to the \mathbb{Z} -torsion submodules and then to the p -primary torsion, and then inducts on n , where p^n is the exponent of this subgroup.

Chapter 5:

*page 96 lines 15 to 17: this is not clear if $\mu > 1$, as in general $\Theta A \neq A\Theta$.

page 99 line 17: “(ii)” should be “the second Torres condition”.

§5.6: see *[Hi05] for an account of the singularities of plane curves using little more than [AM].

*page 112 line 4: $\mu(f)$ is called the “Milnor number”, not the “multiplicity”.

*page 112 line 16: insert “for m sufficiently large” before “by Theorem 13.6 of [EN]”.

*page 114 lines -6, -5: Here $R = (t - 1)H$, as in Theorem 5.10.

*page 114 line -5: “ H ” should be “ $H/(t^k - 1)H$ ”.

*page 119 lines 3-8: see *[Po04] for an improvement upon [MM82], using Reidemeister-Franz torsion.

page 120, line -5: $t - t^{-1}x$ should be $(t - t^{-1})x$.

page 124 line 10: we may delete “If $\rho \neq \varepsilon_F$ ”, for otherwise f and f_0 factor though π/π' , and so are equal.

*page 124 line 12-13: The function is not onto if $\mu = 2$. Thus “if $\mu \leq 2$ $\mu \geq 3$.” should read “in the knot-theoretic case [deR67]. However this map is no longer onto if $\mu \geq 2$.”

§5.10: this section can be substantially improved. The key point is that $V \otimes_K - = W \otimes_\pi -$, where $W = V \otimes_K R[\pi]$ is an $(R[\pi/K], R[\pi])$ -bimodule, and is finitely generated and free as a left $R[\pi/K]$ -module. Hence twisted invariants may be derived from cohomology with local coefficients. From this point of view it is easy to show that twisted Alexander polynomials of symmetric knots satisfy a Murasugi formula *[HLN06].

page 125 line -11: “The elementary” should be “When $C_ = C_*(\tilde{X})$ the elementary”.

page 125 line -8: the notation $H_(X; \alpha, V)$ was not defined. It is of course the homology of the complex $V \otimes_{R[K]} C_*(\tilde{X})$.

page 127 line 15: “coovering” should be “covering”.

Chapter 6:

*page 134 line-10: “Theorem 7 of [Mat]” should be “Theorem 29 of [Mat]”.

*page 143 line -7: insert “ $\varepsilon(\mathbb{D})$ ” between “ \mathbb{D}^{-1} ” and “ u ”.

*page 143 line -1: “ $T_i(s, s)$ ” should be “ $T_i(s^{e(i)}, s)$, for some $e(i) = \pm 1$ and”.

*page 145 lines 4 and 5: these should read

$$\begin{aligned} &= \Sigma \Sigma [\mathbb{D}^{-1}]_{kj} \delta_{ik} \delta_{n0} e(i) t^n \\ &= -[\mathbb{D}^{-1}]_{ij} e(i) \end{aligned}$$

page 147 line 6: “Theorem 1” should be “Theorem 6.1”.

page 147 line -9: “Lemma 11” should be “Lemma 6.12”.

page 147 line -7: “ M_k ” should be “ M_m ”.

Chapter 7:

*page 150, line 18: “ $T_i(s, s)$ ” should be “ $T_i(s^{e(i)}, s)$, for some $e(i) = \pm 1$ and”.

§7.6 should be clarified. In particular the auxiliary space W is used only to specify a framing. The Kervaire obstruction is always 0 *[Da05].

page 165 line -13: “ $N(H)$ ” should be “ $N(Ho)$ ”.

page 165 line -10: “ $X(H)$ ” should be “ $X(Ho)$ ”.

page 166 line 4: the notation is ambiguous; here $\Lambda_2 = \mathbb{Z}[\pi_1(T)]$.

*page 166 line -2: “ $P \cup W$ ” should be “ $P \cup_{X(Ho)} D^4$ ”.

page 167 line 2: delete “(in $E \subset W$)”.

page 167 line 4: “one” should be “on”.

Chapter 8:

*page 172 line -3: insert “not” before “invertible”.

page 174 line 20: if K is strongly $-$ amphicheiral $\Delta_1(K)(t^2) = f(t)f(-t)$ by the argument of Theorem 8.16, and $f(t) \doteq \overline{f(t)} = f(-t^{-1})$ by duality.

page 174 line -13: delete “it”.

page 179 line -7: insert “the” before “covering”.

page 181 lines 5, 14: “ $H_1(X; c^* \Lambda_\mu)$ ” should be “ $H_1(X^\gamma; \mathbb{Z})$ ”.

page 182 lines 8,9,-6,-3 and page 183 line 2: “ λ ” should be “ ℓ ”.

page 182 line 13: Λ should be Λ_2

page 183 lines 11 and 14: “ K ” should be “ $K_{m,n}$ ”.

page 184 line -9: “ $\widehat{\delta}$ ” should be “ $\widehat{\Delta}$ ”.

*page 187 lines 1 and 2: should read “The Alexander polynomial of $\#^3 4_1$ satisfies the Murasugi conditions with $\ell = 1$. This knot”.

*page 187 line 3: “8.8” should be “8.10”.

page 189 Cor. 8.13.1: this is also an easy consequence of [Ed84].

page 190 lines -6, -4: The initial letters of these sentences should be capitalized.

page 197 line -2: “ $A \cap \mathcal{K}$ ” should be “ $A \cap \mathcal{L}$ ”.

page 198 line 18: “.” should be “,”.

*page 199 line -5: The argument of Theorem 8.22 may be refined to show that the denominator divides a power of q *[Hi04]. In fact we may assume that the denominator is 1 *[Ch04]. Cha observes that $\bar{A} \cup \bar{K}$ is concordant to Ho and so the Blanchfield pairing is neutral (without any localization).

Chapter 9:

*page 205 line 5: the result quoted from [FV92] was first proven by Dicks and Sontag, in *[DS78].

page 208, Lemma 9.4: “ N ” should be “ M ” throughout this lemma.

page 208 line -2: “)”) should be “)”.

page 209 line -3: “ $\rightarrow\rightarrow$ ” should be “ \rightarrow ”.

page 211 line 6: close the gap between “exterior” and “.”.

*page 215 Theorem 9.11(6): delete “and only if”. (The *Tor* group could be finite. I know of no such examples).

page 217 line -9: insert a space between “a” and “ $(-1)^q$ -linking”.

*page 218 line 6: “part (4)” should be “parts (3) and (6)”.

Chapter 10:

page 234 line -12:: “minimal model” should be “*minimal model*”.

page 236 line 4: interchange ∂X_1 and ∂X_2 in the equation.

page 237 line -8: “in in” should be “is in”.

page 239 line 18: insert “part (4) of” before “Theorem 10.4”.

page 239 line 22: add “with distinct indices” at the end of this sentence.

*page 239 line -9: “ $\binom{r}{s}$ ” should be “ $\binom{r-1}{s}$ ”.

*page 240 line -12: the result (3) quoted here should read “ $m(I)(\eta_{i_r}) = -m(I)(\eta_{i_1}) = (-1)^r \bar{\mu}(I)$ for $i_1 \neq i_r$, and $m(I)(\eta_j) = 0$ otherwise”.

page 241, line 7: “ $\langle x_i \rangle$ ” should be “ $\langle x_i \rangle$ in $Mil(G)$ ”.

page 241, line 8: “ $G = \Pi A_i$ ” should be “ $Mil(G) = \Pi A_i$ ”.

page 243 line -13: “1-link L_s ” should be “1-links L ”.

page 243 lines -8 to -5: the cited result of Cochran involves a different equivalence relation “ $(2, k)$ -cobordism”.

Chapter 11:

page 247 line -2: “ $n > 1$ ” should be “ $n \geq 1$ ”.

Biblio:

the thesis [She] was published as Memoir 784 of the AMS Memoir (2003).

[COT99] has appeared: Ann. Math. 157 (2003), 433-519.

[Mc01] has appeared: Ann. E.N.S. 35 (2002), 153-171.

[SW02] has appeared: Topology 41 (2002), 979-991.

Add:

- *[Ch04] Cha, J.C. A characterization of the Murasugi polynomial of an equivariantly slice knot, arXiv:math.GT/0404403 v1 22 April 2004.
- *[Da05] Davis, J.F. A two component link with Alexander polynomial one is concordant to the Hopf link,
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- *[DS78] Dicks, W. and Sontag, E. D. Sylvester domains,
J. Pure Appl. Algebra 13 (1978), 243-275.
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- *[Hi05] Hillman, J.A. Singularities of plane algebraic curves,
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- *[HLN06] Hillman, J.A., Livingston, C. and Naik, S. Twisted Alexander polynomials of periodic knots, Alg. Geom. Top. 6 (2006), 143-167.
- *[Po04] Porti, J. Mayberry-Murasugi’s formula for links in homology 3-spheres,
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- *[Yj64] Yajima, T. On simply knotted spheres,
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