Errata and Addenda for the 2007 revision of "Four-Manifolds, Geometries and Knots"

4 June 2009

Chapter 1:

page 4, line -3: "ϕ" should be "ϕ^e".

page 23, line 3: "H^n)" should be "H^n(".

Chapter 2:

page 26, line 10: "$Tor_p^{R[G/H]}(H_q(X;R[G/H]),R[G/J])$" should be "$Tor_p^{R[G/H]}(R[G/J],H_q(X;R[G/H]))$". (The homology modules are defined as *left* modules, while $R[G/J]$ is a bimodule.)

page 29, line 7: This sentence referred to the original version of Theorem 2.4, and should be deleted.

page 29, line 9: "Corollary 8.6 of [Bi]" could be replaced by "Corollary 2.5.1 below".

page 29: Theorem 2.5 is largely redundant, in that it is implied by Corollary 2.4.1 and Theorem 2.8.

page 39: At the end of Theorem 2.14 one could argue somewhat more directly from [Ke83], as in Theorem 7.3, instead of appealing to [Sc83] and [Zi].

Chapter 3: Theorem 3.9: it can be shown that if M is a finite PD_4-space the hypothesis that ν be FP_3 is redundant. (See [Hi08].)

page 66, line 14: Lemma 2.10 and Theorem 3.12 together with the fact that $H^2(\pi;\mathbb{Z}[\pi]) \cong Z^\infty$ may be used to show that there are no such PD_4-complexes P. (See [Hi09].)

Chapter 4:

page 78, line -2: Replace this paragraph by the following simplification. "Since M is aspherical if and only if M_ν is aspherical, (3) follows from (2) and the facts that PD_3-groups have one end and a PD_3-space is aspherical if and only if its fundamental group has one end."

Chapter 5:

page 94, after Cor. 5.6.1.: Add "See §14 of Chapter V of [BPV] for examples of type III admitting at least two such subgroups."

page 98, Lemma 5.11: "by Theorem 5.10" should be "by Lemma 5.9".

page 101, line 10: (in Lemma 5.15) "F^+" should be "f^+".

page 104, Theorem 5.19(2): "$w_1(M) \neq 0$" should be "$w_1(M) = c_{M*}w_1(F)$".

Chapter 6:

page 117, line -10: "id_E" should be "id_M".

Chapter 7:

page 131, lines 11-13: "§2" and "§3" should be "§2 and §3" and "§4", respectively.

page 141, line 11+: every Nil^3 manifold is a cusp of an $\mathbb{H}^2(\mathbb{C})$-manifold [McR09].

page 144, lines 24-25: if a 3-manifold M fibres over S^1 and $\beta_1(M) > 1$ then M has fibrations with fibre of arbitrarily high genus [Bu07].

page 148, line 4: rational surfaces may also have more than one minimal model.

Chapter 9:

page 189: the example following Corollary 9.8.1 is not correct. In fact $t(x^{-1}, 1)$ has order 2 and fixes points of the form $(x.h, h) \in H^2 \times H^2$. However one may construct related examples which are torsion free extensions of $Z/4Z$ by $K \times K$.

Chapter 10:

page 214, line -11: "subgroup" should be "subset". As a group $E_\pi(L) \cong H^2(\pi; \mathbb{Z}^u) \rtimes \{\pm 1\}$.

The statement of Theorem 10.17 is not correct; an (implicit) quantifier over certain elements of $H^2(X; \mathbb{Z}^u)$ is misplaced and should be "there exists" rather than "for all". Where it has

"*and let* $x \in H^2(X; \mathbb{Z}^u)$ *be such that* $(x \cup c_X^* \omega_F)[X] = 1$. *Then there is a* 2*-connected degree-*1 *map* $h : X \to E$ *such that* $c_E = c_X h$ *if and only if* $(c_X^*)^{-1} w_1(X) = (c_E^*)^{-1} w_1(E)$, $[x]_2^2 = 0$ *if* $v_2(E) = 0$ *and* $[x]_2^2 = [x]_2 \cup c_X^*[\omega_F]_2$ *otherwise*"

it should read

"*Then there is a* 2*-connected degree-*1 *map* $h : X \to E$ *such that* $c_E = c_X h$ *if and only if* $(c_X^*)^{-1} w_1(X) = (c_E^*)^{-1} w_1(E)$ *and there is an* $x \in H^2(X; \mathbb{Z}^u)$ *such that* $(x \cup c_X^* \omega_F)[X] = 1$, *with* $[x]_2^2 = 0$ *if* $v_2(E) = 0$ *and* $[x]_2^2 = [x]_2 \cup c_X^*[\omega_F]_2$ *otherwise*".

The argument is otherwise correct. Thus the minimal model Z is uniquely determined by X if and only if $v_2(X)$ is in the image of $H^2(\pi; \mathbb{F}_2)$.

page 216, line 16: If $v_2(X)$ is not in the image of $H^2(\pi; \mathbb{F}_2)$ the minimal model is not unique.

Chapter 11:

page 227, line 17: The reference "[DM85]" should be "[HM86]".

page 227, line 18: No. A group of symmetries of a Brieskorn homology sphere must fix the exceptional fibres and so must be cyclic.

page 228, lines 18-20: Davis and Weinberger have confirmed that these k-invariants agree, if M is *non*orientable.

Chapter 13:

page 252, lines 3, and -2, page 253, line -9 to page 254, line 4, and page 254, lines 14-15: The claim re decompositions is wrong. Let X be the exterior of the figure eight knot, which is a fibred $\mathbb{H}^3$-manifold with fibre the punctured torus. Let ϕ be the self homeomorphism of $\partial X \times S^1 = S^1 \times S^1 \times S^1$ which preserves the longitude of the knot and swaps the meridian and the third factor. Then $M = X \times S^1 \cup_\phi X \times S^1$ fibres over T with fibre $T\#T$ and θ is injective. It is not geometric, but is the union of two pieces of type $\mathbb{H}^3 \times \mathbb{E}^1$.

page 254, line -2: "$[\pi : C_\pi(\phi)]$" should be "$[\pi : \phi C_\pi(\phi)]$".

Re the discussion at the end of §2. Brian Bowditch has shown that for any given base and fibre genera there are only finitely extensions of PD_2^+-groups by PD_2^+-groups which contain no noncyclic abelian subgroups [Bo07].

page 255, line -1: the PD_2^+-group of genus 4 embeds in the genus-2 mapping class group [LR06].

Example 3 of §3 does in fact have a geometric decomposition, as do all torus bundles over hyperbolic surfaces, and "most" Seifert fibred 4-manifolds. I overlooked the possibility of a nonseparating cusp!

Chapter 14:

page 267, line -8: "$rK = r_{n+2}K$, $K\rho = Kr_n$" should be "$K\rho = Kr_n$, $rK = r_{n+2}K$".

page 272, line 17: delete "dividing".

page 276, line 8: the notion of "meridianal automorphism" is defined at the beginning of the next section!

Chapter 16:

page 314, line -9: "$h(G)$" should be $h(\sqrt{G})$".

page 320, line 15: add "and since $[yx^{-1}, x^{-1}y] = h^{3e-2-\eta}$ we find that $q = 3e - 2 - \eta$, and so q is odd" to the end of this sentence.

page 320, line -10: k does not lift to an automorphism of π' if $\eta = -1$, and then $Aut(\pi') \neq Aut(G)$. However the statement of the theorem is still correct.

Chapter 17:

page 330, lines 17-18: the 2-knot with group Φ is unique up to TOP isotopy and reflection. (See [Hi09].)

page 331, lines 1-3: in fact the relevant invariant is the *conjugacy class* of the image of the characteristic map of the fibration in $(Z/2Z)^r \rtimes Aut(F(r))$. If two elements of this semidirect product determins the same meridianal automorphism they are conjugate. Hence the knot group π determines the homotopy type of $M(K)$, and also the homeomorphism type if K is fibred.

Chapter 18:

page 340, line 2: "$\widetilde{M}$" should be "$\widehat{M}$".

page 343, line 6: A proof of this claim was overlooked. See [Pl86]. (The argument on page 352 for branched covers of hyperbolic knots can be adapted to this case.) In fact no 2-knot with group $G(\pm)$ is reflexive and only six admit orientation changing symmetries. (JAH – in preparation.)

page 346, line 24: the definition of Υ is incomplete; the entry in the R^2 factor should be $(1, 0)$.

page 346, line -9: the matrix $\gamma(u)$ should be transposed.

page 346, line -3: "$diag[-1, 1, -]$" should be "Ω_1".

Additional references:

[Bo07] Bowditch, B.H. Atoroidal surface bundles over surfaces, preprint.

[Bu07] Button, J.O. Mapping tori with first Betti number at least two, J. Math. Soc. Japan 59 (2007), 351–370.

[HR05] Hillman, J.A. and Roushon, S.K. Surgery on $\widetilde{\mathbb{SL}} \times \mathbb{E}^n$-manifolds, Canada. Math. Bull., to appear.

[Hi09] Hillman, J.A. Strongly minimal PD_4-complexes, Top. Appl., to appear.

[Hi08] Hillman, J.A. Finiteness conditions in covers of Poincaré duality spaces, preprint.

[LR06] Leininger, C. J. and Reid, A. W. A combination theorem for Veech subgroups of the mapping class group. Geom. Funct. Anal. 16 (2006), no. 2, 403–436.

[McR09] McReynolds, D.B. Controlling manifold covers of orbifolds, arXiv:0901.4239v1 [math.GT].