Errata and Addenda for the 2014 revision of
“Four-Manifolds, Geometries and Knots”

1 April 2015

Starred references refer to new items in the bibliography below.

Fonts: The “blackboard bold” font has been used
(1) for the standard coefficient rings \( \mathbb{Z}, \mathbb{Q}, \mathbb{R} \) and \( \mathbb{C} \);
(2) to distinguish geometries \( X \) from their associated model spaces \( \hat{X} \);
(3) to distinguish the (based) infinite cyclic group \( \mathbb{Z} \) from unbased infinite cyclic groups \( \mathbb{Z} \); and
(4) (in \( \S 2.9 \) and Theorem 4.1 only) to denote hyper(co)homology.

However, these principles have not been applied consistently. In particular, the blackboard bold font should have been used for twisted integer coefficient modules \( \mathbb{Z}_u \) in (co)homology groups, and also for vector spaces \( \mathbb{R}^n \) and \( \mathbb{C}^n \). I have not attempted to list all the occasions in which the font should have been changed.

Chapter 1:
page 4, line 6: add “of finite index” after “subgroups”.
page 7, line −4: “\( \mathbb{Z} \times \Gamma_q \)” should be “\( \Gamma_q \times \mathbb{Z} \).”

Chapter 2:
page 40, line −11: “c.d.U_i” should be “\( c.d.\hat{U}_i \)”.
page 41, lines 21-22: if \( \chi(H) < 0 \) this corollary follows quickly from the finite divisibility of \( \chi(H) \).

Chapter 3:
page 48, line −7: “\( H_3(D_*) \otimes_{\mathbb{Z}[\pi]} \mathbb{Z} \)” should be “\( \mathbb{Z} \otimes_{\mathbb{Z}[\pi]} H_3(D_*) \)”.
page 55, line 23: “[Hi08]” should be “[Hi13b].”
page 66, lines 20-24: in the orientable case \( w_2 \) suffices [Hi13c, HKT09*].
page 67, line 13: In Lemma 3.18(5), add “\( \pi \) is infinite”. (In fact \( |\pi| > 2 \) suffices.)

Chapter 4:
page 70, line 22: the final comma should be a full stop.
page 70, line 23: add the sentence
“If \( G \) is a group and \( A \) is a left \( \mathbb{Z}[G] \)-module let \( |A| \) be the \( \mathbb{Z}[G] \)-module with the same underlying group and trivial \( G \)-action.”.
page 80, Corollary 4.3.3 et seq.: the homotopy types of such mapping tori are determined by \( \pi, w_1(M) \) and the orbit of \( w_2(M) \) under the action of \( \text{Out}(\pi) \). See Theorem 29 of [Hi13c].

Chapter 5: page 93, line 2: “[\( \pi/K : p(C_\pi(K)) \])” should be “[\( \pi/K : p(C_\pi(K)) \])”.
page 94, Corollary 5.6.1: N.Salter has shown that for each \( n \geq 1 \) there are such groups \( \pi \) with \( \chi(\pi) = 24n - 8 \) which have at least \( 2^n \) distinct subgroups \( K \) such that \( K \) and \( \pi/K \) are orientable [Sa14*].
(On the other hand, Cor. 5.6.1 leads to an upper bound of \( d^{2d+3} \), where \( d = \frac{\chi(\pi)}{4} \). Moreover, if \( \chi(\pi) \geq 16 \) then at most \( 2^{x(\pi)} \) of these have \( |\chi(\pi/K)| > \log \chi(\pi) \).)

Chapter 6:
page 113, lines 9-10: this clause should read
“surfaces with nonempty boundary other than \( D^2 \) or the annulus”.
page 121, lines −5, −4: “\( D \)” should be “\( \mathcal{D} \)”.
page 124, lines 7 and 8: replace this sentence by
Closed orientable 4-manifolds \( M \) with \( \chi(M) = 0 \) and \( \pi \cong \mathbb{Z}_*m \) are also determined up to homeomorphism by \( \pi \) and \( w(M) \) \cite{Hi09,HKT09*}.

**Chapter 7:**
- page 144, line −5: “9.8” should be “9.9”.
- page 176, lines -11 to -6: The sentence beginning “No other bases ...” and the material in parentheses is wrong. In fact, \( P(2,2) \) and \( D(2,2) \) do occur as Seifert bases of \( \text{Sol}^3 \times \mathbb{E}^1 \)-manifolds. The preprint \cite{Hi13d} is seriously flawed, and is being rewritten, as of April 2015.

**Chapter 9:**
- page 179, line 16: “Little” should be “Relatively little”.
- page 179, lines 21-23: delete the qualifications “orientable” and “(excepting ... \( \mathbb{SL} \times \mathbb{E}^1 \))”.
- page 182, line 16: add “, which is Corollary 9.6.2 below” after “[Ke]”.
- page 192, line -3: “almost coherent. (See §1” should be “coherent. (See §4”.

**Chapter 10:**
- page 199, line 11: the sentence should end with a fullstop.
- page 216, line 2: delete “and only if”.

**Chapter 12:**
- pages 241–244: “\( \Gamma(\Pi) \otimes_{\mathbb{Z}[\pi]} \mathbb{Z}^w \)” should be “\( \mathbb{Z}^w \otimes_{\mathbb{Z}[\pi]} \Gamma(\Pi) \)” throughout §12.6.

**Chapter 13:**
- page 259, line 13: insert “with \( \chi(E) > 0 \)” after “E”.

**Chapter 14:**
- page 279, line 14: the smallest possible HNN base is \( Q(16) \) \cite{BH13*}.
- page 293, line -6: “, the” should be “, this”.

**Chapter 16:**
- page 311, line 11: replace “most such” by “all such”.
- page 316, line 23: “\( \mathbb{N} \)” should be “\( \mathbb{N} \)”.
- page 319, line 1 of text: “\( h(\sqrt{\pi K}) \)” should be “\( h(\sqrt{\pi K}) \)”.

**Chapter 17:**
- page 328, line 16: replace “[Hi09]” by “. (See §17.6 below.)”.

**Chapter 18:**
- page 356, lines 3-10: the formulation of twist spinning used here is from \cite{Ze65}.
- page 356, line 8: “\( (S, \theta) \)” should be “\( (s, \theta) \)”.
- page 356: expand the final two sentences of the proof of Theorem 18.14 as follows: “Let \( \alpha \) be the automorphism of \( \pi \) given by \( \alpha(t) = t^{-1}, \alpha(x) = x \) and \( \alpha(y) = y \). If there were an automorphism \( \theta \) such that \( \theta(u^n t) = (u^n t)^{-1} \) then \( \alpha c \theta(u^n t) = u^{-n} t \). Hence if \( n \neq 0 \) then \( K \) is not \( +\)amphicheiral.”
Bibliography: The following list includes the items cited from the arXiv in the 2014 revision and new references. The details shall be updated when possible.

References


[Ha13] Hamenstädt, U. On surface subgroups of mapping class groups, video lecture, MSRI, 22 March 2013.


[Hi13d] Hillman, J.A. $\mathbb{Sol} \times \mathbb{E}^2$-manifolds, arXiv: 1304.2436 [math.GT].


[Lo13] is now [LM14] below (with the same arXiv identifier).


