Errata and Addenda for the 2014 revision of
"Four-Manifolds, Geometries and Knots"

16 July 2018

Starred references refer to new items in the bibliography below.

Fonts: The "blackboard bold" font has been used

(1) for the standard coefficient rings $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$;
(2) to distinguish geometries $\mathbb{X}$ from their associated model spaces X;
(3) to distinguish the (based) infinite cyclic group $\mathbb{Z}$ from unbased infinite cyclic groups Z; and
(4) (in §2.9 and Theorem 4.1 only) to denote hyper(co)homology.

However, these principles have not been applied consistently. In particular, the blackboard bold font should have been used for twisted integer coefficient modules $\mathbb{Z}^u$ in (co)homology groups, and also for vector spaces $\mathbb{R}^n$ and $\mathbb{C}^n$. I have not attempted to list all the occasions in which the font should have been changed. (The changes have been made in the version on my home page.)

I have also replaced $\leq$ and $\geq$ by $\leqslant$ and $\geqslant$ (respectively) throughout.

Chapter 1:

page 4, line 6; add "of finite index" after "under taking subgroups".

page 7: A line dropped out of the first sentence of the proof of Theorem 1.5, in an earlier revision. "torsion - abelian" should be "torsion-free nilpotent, by Proposition 5.2.19 of [Ro]. We may assume that G is not abelian".

page 7, line -4: "$Z \times \Gamma_q$" should be "$\Gamma_q \times \mathbb{Z}$".

§1.7: In several places, when considering the E^2 page of an LHS spectral sequence (e.g., in Theorems 2.12 and 8.1), I have used without comment the fact that if M is a left $\mathbb{Z}[G]$-module and $M|_1$ is the underlying abelian group then $M \otimes \mathbb{Z}[G]$ (with the diagonal G-action) is canonically isomorphic to the induced module $M|_1 \otimes \mathbb{Z}[G]$. Hence $H^p(G/N; H^q(N; \mathbb{Z}[G])) \cong H^p(G/N; \mathbb{Z}[G/N]) \otimes H^q(N; \mathbb{Z}[N])$, if G is FP_p and N is FP_q. This should have been stated here. (See Lemma 5.6 of [Bi].)

page 18, lines 7-9: replace "and since ... contradicts" by "$nn' \in N$ also. But $nn' = ngng^{-1}$ has infinite order in G, by".

page 22, line 11: add "Let $\sigma : G \to \pi$ be a (setwise) section of the projection".

Chapter 2:

page 34, lines 11-14: if X and Y are PD_n-complexes and Y is non-orientable then a map $f : X \to Y$ such that $f^* w_1(Y) = w_1(X)$ only determines a homomorphism from $H_n(X; \mathbb{Z}^{w_1(X)})$ to $H_n(Y; \mathbb{Z}^{w_1(Y)})$ *up to sign*, as one must choose a lift $f^+ : X^+ \to Y^+$. A similar issue arises with other non-constant local coefficient systems. (See [Ta08]* for a thorough discussion of the subtleties here. This oversight has no serious consequences for the present work.)

page 37, line 5: "$X = P$" should be "$X = P_{\langle n \rangle}$".

page 39, line -13: "$c.d\sqrt{G}$" should be "$c.d.\sqrt{G}$".

page 40, line -11: "$c.dU_i$" should be "$c.d.U_i$".

page 41, lines 21-22: if $\chi(H) < 0$ this follows from the finite divisibility of $\chi(H)$.

page 43, line -11: "[KK05]" should be "[Ca07]".

page 44, middle: the identification of $k_1(X)$ with an iterated extension class deserves comment, as I have not found a published proof. We may construct a $K(\pi, 1)$ by adjoining cells of dimension ≥ 3 to X, and $k_1(X)$ is then the primary obstruction to retracting $K(\pi,1)$ onto X. An application of the Homotopy Addition Theorem (see §7.5 of [Sp]) to the standard 3-cocycle representing $k_1(X)$ shows that it is the composite of the projection of the 2-cycles Z_2 onto $H_2(X;\mathbb{Z}[\pi])$ with the inverse of the Hurewicz isomorphism, and so represents the class of the extension.

If P is aspherical no additive natural transformation takes $k_1(P)=0$ to $[P]$.

Chapter 3:

page 48, line -7: "$H_3(D_*)\otimes_{\mathbb{Z}[\pi]} Z$" should be "$\mathbb{Z}\otimes_{\mathbb{Z}[\pi]} H_3(D_*)$".

page 49, lines 1-2: the text should be in *slanted* font.

page 49, lines -9 and -3: "Weak" should be "Strong" [Bass Conjecture].

page 52, Theorem 3.8: the triple $(P_2(M), w_1(M), f_{M*}[M])$ is a complete invariant of the homotopy type, for M a PD_4-complex [BB08]*. However which such triples are thus realized is not known.

page 55, line 23: "[Hi08']" should be "[Hi13b]".

page 66, lines 20-24: in the orientable case w_2 suffices [Hi13c, HKT09].

page 67, line 13: In Lemma 3.18(5), add "π is infinite". (In fact $|\pi|>2$ suffices.)

Chapter 4:

page 70, line 22: the final comma should be a full stop.

page 71, line 20: "$(|\mathbb{Z}[G]|^G)^n$" should be "$\mathbb{Z}[G]^n|_1{}^G$".

page 80, Corollary 4.5.3 *et seq.*: the homotopy types of such mapping tori are determined by π, $w_1(M)$ and the orbit of $w_2(M)$ under the action of $Out(\pi)$. See Theorem 35 of [Hi13c].

Chapter 5: page 93, line 2: "$]\pi/K : p(C_\pi(K)]$" should be "$[\pi/K : p(C_\pi(K)]$".

page 94, Corollary 5.6.1: N.Salter has shown that for each $n\geq 1$ there are such groups π with $\chi(\pi)=24n-8$ which have at least 2^n distinct subgroups K such that K and π/K are orientable [Sa15]*.

(On the other hand, Cor. 5.6.1 leads to an upper bound of d^{2d+3}, where $d=\frac{\chi(\pi)}{4}$. Moreover, if $\chi(\pi)\geq 16$ then at most $2^{\chi(\pi)}$ of these have $|\chi(\pi/K)|>\log\chi(\pi)$.)

Chapter 6:

page 113, lines 9-10: this clause should read

"surfaces with nonempty boundary other than D^2 or the annulus".

page 121, lines $-5,-4$: "D should be "$\mathcal{D}$".

page 124, lines 7 and 8: closed orientable 4-manifolds M with $\chi(M)=0$ and $\pi\cong Z*_m$ are also determined up to homeomorphism by π and $w(M)$ [Hi09,HKT09].

Chapter 7:

page 141, lines 17 to -10: add ", and M is aspherical" to first sentence, delete the rest of the paragraph, and close the gap to the next paragraph.

page 144, line -5: "9.8" should be "9.9".

Chapter 8:

page 161, line -2: we should have observed here that if G is orientable and J and $\tilde{J}$ are both non-orientable then $J*_\phi\tilde{J}$ is orientable, and so has infinite abelianization.

page 162, line 22: The last relation in the presentation given for $G_2 *_\phi B_2$ is redundant, and so this group has a 2-generator presentation of deficiency 0.

page 176, lines -11 to -5: The sentence beginning "No other bases ..." and the material in parentheses is wrong. In fact, P(2,2) and $\mathbb{D}(2,2)$ do occur as Seifert bases of $\mathbb{S}ol^3\times\mathbb{E}^1$-manifolds. The original version of [Hi13d] was seriously flawed; a revision has appeared, as [Hi18].

Chapter 9:

page 179, line 16: "Little" should be "Relatively little".

page 179, lines 21-23: delete "orientable" and "(excepting … $\widetilde{\mathbb{SL}} \times \mathbb{E}^1$)".

page 181, lines 6-7: yes, $[\pi : \rho] = \infty$ suffices here. See [Hi17]*.

page 182, line 16: add ", which is Corollary 9.6.2 below" after "[Ke]".

page 182, line 22: Proposition 8.27 of [Rg] needs also that compact factors of $G_o/Rad(G)$ act nontrivially on $Rad(G)$ [Ge15]*. This holds in Theorem 9.3.

page 192, line -3: "almost coherent. (See §1" should be "coherent. (See §4".

page 193, line -12: add "(See [BJS17].)".

Chapter 10:

page 199, line 11: the sentence should end with a fullstop.

page 216, line 2: delete "and only if".

Chapter 11:

page 221, line -2, to page 222, line 3: the details need some correction, as powers of z^3 were overlooked (when $k > 1$). The strategy and conclusion are correct.

page 222, lines 5 and 6: the presentation for I^* should be $\langle y, z \mid (yz)^2 = y^3 = z^5 \rangle$.

page 222, line 6: "x, y to $\left(\begin{smallmatrix} 2 & 0 \\ 1 & 3 \end{smallmatrix}\right)$ and $y =$" should be "y, z to $\left(\begin{smallmatrix} 2 & 2 \\ 1 & 4 \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} 4 & 1 \\ 0 & 4 \end{smallmatrix}\right)$".

page 222, line 20: "x^3y" should be "z^4".

page 232, line 6: insert "(except in the final sentence)" after "orientable".

Chapter 12:

page 238, in §4, at the end of the subsection on $\pi = Z/2Z$: add

"In each case, $xSq^1u = Sq^1(xu) + x^2u = 0$, and so $Sq^1u = 0$.".

pages 241–244: "$\Gamma(\Pi) \otimes_{\mathbb{Z}[\pi]} Z^w$" should be "$\mathbb{Z}^w \otimes_{\mathbb{Z}[\pi]} \Gamma(\Pi)$" throughout §12.6.

Chapter 13:

page 258, line -10: insert "> 0" after "Euler characteristic".

page 258, line -5: insert "[Re06]" after M_2.

page 259, line 13: insert "with $\chi(E) > 0$" after "E".

page 259, line 15: add "There are non-geometric examples. One with B and F of genus 2 and $\mathrm{Im}(\theta) \cong D$ is given in [Hi11]." to this paragraph.

page 259, line 18: "G" should be "$\mathcal{G}$".

Chapter 14:

page 279, line 14: the smallest possible HNN base is $Q(16)$ [BH17]*.

page 281, lines -9 to -7: replace: "finitely … and 16.3" by "$\mathbb{Z} \oplus (Z/2Z)$ or is finite. (See Theorem 15.8". [The results of Chapter 15 have been improved.]

page 293, line -6: ", the" should be ", this".

Chapter 15:

page 295, line -1: "central subgroups" should be "copies of $\mathbb{Z}$".

page 299, line 13: "Theorem 15.3" should be "Theorem 15.5".

page 300, line 5: "rank" should be "Hirsch number".

page 301, lines 1-2: this sentence should read

"The five possibilities in case (2) derive from Theorem 15.5".

page 301, lines 7 and 17; "Lemma 15.6" should be "Lemma 15.7".

page 302, lines 2-3: yes, $\sqrt{\pi}$ is nilpotent, for all 2-knot groups π. See [Hi17]*.

page 302, Corollary 15.8.2: insert "is" after "If π'".

page 302, line -5: "$\zeta\pi \cong \mathbb{Z}$" should be "$\zeta\pi' \cong \mathbb{Z}$".

page 302, line -3: the relations should read "$ta_it^{-1} = b_{n-i},\ tb_it^{-1} = a_{n-i}b_{n-i}$".

page 305, lines -11 and -6: “[Yo82]” should be “[Yo80]”.

page 306, lines 10-21: this paragraph is based on a wrong presentation for I^*. (Such groups have 3-generator presentations of deficiency -1, if $n > 1$.)

[Chapter 15 has since been completely rewritten, to incorporate [Hi17]*.]

Chapter 16:

page 311, line 11: replace “most such” by “all such”.

page 316, line 2: insert “(up to Gluck reconstruction)” after “is”.

page 316, line 23: “N” should be “N”.

page 319, line 1 of text: “$h(\sqrt{\pi K}$” should be “$h(\sqrt{\pi K})$”.

Chapter 17:

page 328, line 16: replace “[Hi09]” by “. (See §17.6 below.)”.

page 335, line 16: insert “with 1-connected boundary” after “5-manifold”.

page 340: A phrase dropped out of the proof of Theorem 17.14, in an earlier revision. Insert “[Wl86].) Since π meets” between “3.3 of” and “$\zeta Isom_o(\mathbb{SL} \times \mathbb{E}^1)$”.

Chapter 18:

page 356, lines 3-10: the formulation of twist spinning used here is from [Ze65].

page 356, line 8: “(S, θ)” should be “(s, θ)”.

page 356: expand the final two sentences of the proof of Theorem 18.14 as follows

“Let α be the automorphism of π given by $\alpha(t) = t^{-1}$, $\alpha(x) = x$ and $\alpha(y) = y$. If there were an automorphism θ such that $\theta(u^n t) = (u^n t)^{-1}$ then $\alpha c_t \theta(u^n t) = u^{-n} t$. Hence if $n \neq 0$ then K is not +amphicheiral.”

Bibliography: The following list includes the items cited from the arXiv in the 2014 revision and new references. The details shall be updated when possible.

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