

**Errata and Addenda for the 2018 revision of  
“Four-Manifolds, Geometries and Knots”**

15 October 2018

**Chapter 2:**

page 27, line 7: the “Euler characteristic formula” is  $\chi(X) = \sum (-1)^i \beta_i^{(2)}(X)$ ,  
of course.

page 34, line 15: This sentence could be expanded out as follows:

If  $X$  and  $Y$  are  $PD_n$ -complexes and  $Y$  is non-orientable then a map  $f : X \rightarrow Y$  such that  $f^*w_1(Y) = w_1(X)$  only determines a homomorphism from  $H_n(X; \mathbb{Z}^{w_1(X)})$  to  $H_n(Y; \mathbb{Z}^{w_1(Y)})$  up to sign, as one must choose a lift  $f^+ : X^+ \rightarrow Y^+$ . A similar issue arises with other non-constant local coefficient systems. (See [Ta08] for a thorough discussion of the subtleties here. This oversight has no serious consequences for the present work.)

page 36, Theorem 2.11: the relevant action used here is the natural right action on  $Ext_{\mathbb{Z}[\pi]}^1(\mathbb{Z}, \mathbb{Z}[\pi])$ , NOT the action via conjugation considered in Chapter III.§8 of [Br].

page 44, middle: the identification of  $k_1(X)$  with an iterated extension class deserves comment, as I have not found a published proof, although I suspect one may be found in the work of Eilenberg, Mac Lane and J.H.C. Whitehead of the late 1940s. We may construct a  $K(\pi, 1)$  by adjoining cells of dimension  $\geq 3$  to  $X$ . An application of the Homotopy Addition Theorem (see §7.5 of [Sp]) to the standard 3-cocycle representing the primary obstruction to retracting  $K(\pi, 1)$  onto  $X$  shows that it is the composite of the projection of the 2-cycles  $Z_2$  onto  $H_2(X; \mathbb{Z}[\pi])$  with the inverse of the Hurewicz isomorphism, and so represents the class of the extension.

If  $P$  is aspherical no additive natural transformation takes  $k_1(P) = 0$  to  $[P]$ .

**Chapter 11:**

§11.2: The groups  $P''_{48.3^{k-1}a} \times Z/dZ$  were overlooked here! This oversight has yet to be corrected. It has no consequences for other chapters. In particular, these groups cannot be knot commutator subgroups, as they have abelianization  $Z/2dZ$ .

**Chapter 15:**

page 299: “ $g \in \pi^+$ ” should be “ $w_1(X)(g) = 1$ ”, as the notation  $\pi^+$  has not been defined.

page 306, line -4: “15.14” should be “15.12”.

**Chapter 18:**

page 364, line -4: “Chapter 12” should be “Chapters 8, 12 and 13”.

**Bibliography:** The following list includes the items cited from the arXiv in the 2018 revision (but not yet published elsewhere) and new references. The details shall be updated when possible.

## REFERENCES

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- [Ka98] Kapovich, M. On normal subgroups in the fundamental groups of complex surfaces, preprint, University of Utah (August, 1998).  
(This does not seem to have been published.)
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