Errata and Addenda for "Four-Manifolds, Geometries and Knots"

6 March 2007

A revision is now available, including relaxation of finiteness conditions (notably in Chapter 4) and completion of the treatment of the Seifert fibred geometries, as well as the corrections listed below.

Starred references refer to the additional bibliography below.

Chapter 1:

page 5, line -6: "M" should be "it".

page 6, Cor. 1.3.1: "$\mathbb{R}$" should be "$\mathbb{Q}$" throughout.

page 6 line 13: the $(1,1)$ entry of the first matrix should be "2", not "1".

page 9, line -7: Yes. There is a finitely presentable amenable group not in SA. See [BV05]*.

page 11, proof of Theorem 1.8: "LU_α" should be ℓU_α" throughout.

page 11, line -2: "assume" should be "Assume".

page 12, lines 14-15: insert "$M(h+1)$ and derived length at most $d(h+1)$ (since it contains the preimage of" after "at most". (A "\$" was mistyped as a "%" in the .tex file.)

page 12, line -8: "bu" should be "by".

Theorem 1.11: delete "countable"; it is not used in the proof.

page 17, line -4: "base H" should be "base B".

page 18, line -11: delete the first occurance of "and so $H^s(G;\mathbb{Z}[G])=0$ for $s\leq 2$" in this sentence.

page 19, line 17: this holds also if E is torsion-free, elementary amenable and $h(E)>1$, as a consequence of Prop. 3 of [Kr93']*.

page 20, line -7: "Z." should be "Z,".

Chapter 2:

page 26, line -12: "$H^3(\pi,$" should be "$H^3(\pi;$".

Theorem 2.3: "infinite" is redundant.

page 31, line 7: "almost coherent" should qualify both options. (In fact the theorem is correct as it stands, by the result of [Gi79] quoted earlier).

Theorem 2.8: the hypothesis "$c.d.\pi\leq 2$" may be relaxed to "$c.d_{\mathbb{Q}}\pi\leq 2$", since $\pi_2(X)$ embeds in $H_2(\tilde{X};\mathbb{Q})$.

page 32, line 4: the sequence should end $\quad\cdots\to\mathbb{Z}[\pi]\to\mathbb{Z}\to 0$.

In §2.4 "as defined in Chapter 2" should be "as defined in Chapter 1."

page 34, lines -6 to -1: this group satisfies Turaev's criterion. See [Hi06]*.

Theorem 2.13: insert "almost coherent" between "restrained," and "locally virtually indicable" in the first sentence of the proof.

page 38, line -3: the "finite $K(G, 1)$" assumption in [KK99] has been removed in the published version.

Lemma 2.15: in the first sentence "$[G : H]$" should be "$[G : J]$".

At the end of this section add:

"It is possible to use the fact that $Out(H)$ is virtually torsion-free instead of appealing to [GS81] to prove this corollary.".

page 44, line 13: "$2r + s - 1$" should be "$r + s - 1$".

Chapter 3:

Theorem 3.2: a closely related result was given earlier in [Ha87]*.

Theorem 3.6: similarly, if π is any PD_4-group of type FF then
$\mathrm{def}(\pi) < 1 - \frac{1}{2}\chi(\pi)$.

page 52, line -6: "$H^4(f_E; Z^{w_1(E)})$" should be "$H_4(f_E; Z^{w_1(E)})$".

page 54, line -9: "aumentation" should be "augmentation".

page 56, line 6: "$\pi/C_\pi(F)$" should be "$\pi/FC_\pi(F)$" (twice).

Corollary 3.9.3: a version of this result is stated in [Ku78]*. (Kulkarni assumes that π satisfies a homological finiteness condition).

page 56, line -5: "FP" should be "FP_∞".

Theorem 3.12: add "and $w_1(M)$" at the end of the enunciation. (The conjugation switching left and right module structures and used to form duals depends on the orientation character.)

The final paragraph of the proof should be as follows: "If $c.d.\pi \leq 2$ the argument can be modified to work over $\mathbb{Z}[\pi]$. In particular, $B_1 = Z_1$ is projective, so $C_2 \cong Z_1 \oplus Z_2$ and the integral chain complex of $\widetilde{M}$ is chain homotopy equivalent to the sum of a projective resolution of $\mathbb{Z}$ and the dual of a projective resolution of $\mathbb{Z}$ (shifted to terminate in degree 2)."

(It is correct as it stands if π has one end.)

The second sentence of the Addendum to Theorem 3.12 can be strengthened as follows: "Then $S \otimes \pi_2(M) \cong R \oplus \overline{H^2(\pi; S[\pi])}$, where R is a projective $S[\pi]$-module, and $\chi(M) \geq 2(1 - \beta_1(\pi; S) + \beta_2(\pi; S))$, with equality if and only if $\pi_2(M) \cong \overline{H^2(\pi; \mathbb{Z}[\pi])}$."

Corollary 3.13.1: in fact $q(\pi) = 2$, for otherwise $c_M^* H^2(\pi; \mathbb{F}_2)$ would not be self-annihilating under cup product into $H^4(M; \mathbb{F}_2)$.

page 65, line 11: "Since $\widehat{M}$ is a connected open 4-manifold" should be "Since $\widehat{M}$ is connected and G is infinite".

Chapter 4:

page 69, line 6: "0 ." should be "0.".

page 70, Theorem 4.1: delete "almost" in part (3) of the enunciation. (I read too much into [KK99]).

pages 71-72: $\widehat{\mathbb{Z}[\pi]}$ is in fact the module coinduced from $\mathbb{Z}[\pi]|_\nu$, and so for consistency with the usual definition the signs should be changed in some of the exponents in the formulae on page 71, line -6, page 72, lines 2, 7 and -9. (The book follows the conventions of [Ba80'] here. See also page 21).

page 72, line 17: "$H^*(C_*; v)$" should be "$H^*(C_*; \widehat{\mathbb{Z}[\pi]})$".

page 75, lines 2, 9 and 12: "δ" should be "δ_M".

page 75, line 9: "m" should be "p".

Theorem 4.5: if we assume only that ν is finitely *generated* then M' still satisfies Poincaré duality with local coefficients of formal dimension 3.

page 78, line 6: "hence" should be "Hence".

page 78, line 7: "infiite" should be "infinite".

page 78, line -1: such Novikov rings are weakly finite [Ko05].

page 79, line 22: "$Out(F(r)$" should be "$Out(F(r))$".

page 79, line -11: "[He]" should be "[Hn]".

Theorem 4.6: the hypothesis that ν be torsion-free can be removed, since finite presentability of $\pi_2(N)$ follows from Theorem 2.18 and the fact that $\pi_1(N)$ is virtually torsion-free. (Thus Corollary 4.6.2 may be deleted).

Corollary 4.6.1: in the enunciation assume also that $\chi(M) = 0$, and delete "*which is isomorphic to the fundamental group of a 3-manifold N*". (One must consider also the orientations of the factors).

page 83, line 15: "is" should be "be".

page 84, line 7: "3.4" should be "3.5".

Chapter 5:

page 92, line -9: "we must have $\Sigma \leq K_1$" should be "we must have $\Sigma \leq K$".

page 93, line 21: "[Jo98]" should be "[Jo99]".

page 93, lines -5 to -1: In Corollary 5.6.2: delete "$\chi(K) = 0$ or", and delete Corollary 5.6.3. (faulty arithmetic!)

page 95, line 8: "3.4" should be "3.5".

page 102, line 11: "3.11" should be "3.12".

page 102, lines 14-17: Yes. In fact it suffices to adapt Theorem 1.19 to the nonaspherical case.

page 105 lines 11 and 12: delete this paragraph. (The second sentence was wrong, and the hypothesis on π_3 is unnecessary.)

Chapter 6:

page 118, line 16: "[Da95, 6′]" should be "**6.6′** [Da95]".

Lemma 6.9: "I_π^+ is an isomorphism" should be "I_π^+ is an epimorphism". (This is all that is ever needed later.)

Chapter 7:

page 133, line 9: "$\frac{w}{cz+d}$" should be "$\frac{w}{(cz+d)^2}$".

page 135, line 5: "ζ^2" should be "ζ_2".

page 135, lines -7 to -5: see [Ba04]* for a proof (using $\mathbb{R}$-algebraic groups) that compact infrasolvmanifolds (in any dimension) have discrete presentations with finite holonomy and are determined up to diffeomorphism by their fundamental groups, and for a new proof of the main result of [AJ76].

page 137, line 10: "t^{-1}" should be "$-t$".

page 137, line -5: "$\alpha, b, c, d)$" should be "(t, x, y, z)".

page 140, line 20: "[KL96]" should be "[Eb82]"*.

page 143, line -1: "Theorem 7.2" should be "Theorem 7.3".

page 144, line 7: "Theorem 7.2" should be "Theorem 7.3".

§7.7 (pages 144-146): this section should be rewritten. In particular the assertion on lines 6-7 of page 146 should be justified. The foliations of the Lie groups Nil^4, $Nil^3 \times R$ and $Sol^3 \times R$ by cosets of the normal subgroups $\zeta^2 Nil^4$, $\zeta Nil^3 \times R$ and $Sol^{3\prime}$ are preserved by the isometries and so induce Seifert fibrations on quotients by lattices. A different argument is needed in the flat case since not all flat 4-manifolds are Seifert fibred.

Ue's theorem has been extended to nonorientable 4-manifolds [Kemp]*.

Chapter 8:

In §8.3 the argument using the Lefshetz Fixed Point Theorem to eliminate A_4 as a possible holonomy group for a flat 3-manifold is wrong. (My thanks to A.Szczepanski for alerting me to this blunder.) The result is of course well-known to be correct. The paragraph containing this error may be adjusted as follows:

(i) in the third sentence (beginning "Assuming ...") replace the phrase "and a little topology" by "and some calculation";

(ii) replace the final three sentences (beginning "But any ...") by "Suppose that G is a flat 3-manifold group with holonomy A_4. It is easily seen that

G_6 is the only flat 3-manifold group with holonomy $(Z/2Z)^2$, and so we may assume that the images in $SL(3,\mathbb{Z})$ of the elements of order 2 are diagonal matrices. It then follows easily that the images of the elements of order 3 are (signed) permutation matrices. (Solve the linear equations $wu = vw$ and $wv = uvw$ in $SL(3,\mathbb{Z})$, where $u = diag[1,-1,-1]$ and $v = diag[-1,-1,1]$.) Hence G has a presentation of the form

$$\langle Z^3, u, v, w \mid ux = xu, yuy = u, zuz = u, xvx = v, yvy = v, zv = vz, wx = zw,$$
$$wy = xw, wz = yw, wu = vw, u^2 = x, w^3 = x^a y^b z^c, (uw)^3 = x^p y^q z^r\rangle.$$

It may be checked that no such group is torsion-free. Therefore neither A_4 nor S_4 can be the holonomy group of a flat 3-manifold."

page 156, line 22: "Γ_2" should be "Υ_2".

page 166, Theorem 8.4: "an infrasolvmanifold" should be "a closed 4-manifold with a geometry of solvable Lie type".

page 169, line -3: "If $\bar{\theta}$" should be "If $\bar{\theta}^2$".

§8.10 should be updated, in the light of [Ba04]*.

page 175, lines -10,-9: Delete the sentence "Clearly $d' \leq 4 - \beta_1(\sqrt{\pi})$.".

Theorem 8.9: Using [Vo77]* we may relax the restriction on π to "if π is virtually abelian and $\beta_1(\pi) = 1$ assume π orientable".

page 176, line 4: "[DeK]" should be "[De]".

Chapter 9:

page 184, line -8: "$H^*(\pi;\mathbb{Q}[\pi])$" should be "$H^*(\nu;\mathbb{Q}[\nu])$".

page 184, lines -3 to -1: this can be done, and "orientability" is unnecessary.

page 188, line 22: "Theorem 5.6" should be "Theorem 5.5".

page 192, line -9: "[TS01]" should be "[RT01]".

page 193, line 12: delete the second "M" in the first sentence.

page 193, line -11: "[KL96]" should be "[Eb82]"*.

page 193, line -13: This result holds for all $\widetilde{\mathbb{SL}} \times \mathbb{E}^1$-manifolds [HR05]*.

Chapter 10:

Theorem 10.5: this theorem can be improved to show that in all cases there are at most two such homotopy types with given algebraic 2-type.

page 198, line 10: "ρ" should be "K".

page 200, line 21: u and w_1 each split the inclusion of the unique $Z/2Z$ subgroup, but in general $u \neq w_1$.

page 200 line -10: it is *not* true in general that c_M is induced by Bj. (When u is trivial there are two such fibrations.)

page 205, line 9: In the first paragraph of the proof of Theorem 10.9, "$\xi\Omega(h)$" should be "$\xi\hat{\Omega}_*(h)$".

Theorem 10.13: this would be an immediate consequence of Theorems 2.11 and 4.5, provided that we removed "finitely dominated" from the definition of PD_3-complex.

page 212, line 4: Using [Fa77]* we can show that $H^*(Z *_Z D; \mathbb{F}_2) \cong \mathbb{F}_2[S,U,V]/(S^2, SU, SV, V^2 + aU^2V)$ for some $a = 0$ or 1. Lemma 10.15 is still needed to show that $a = 1$.

page 213, line -14: "$k \in K'$" should be "k in the subgroup generated by r^2 and s^2".

page 214, line 14: "ρ" should be "K".

Chapter 11:

page 217, line -2: "π *and*" should be "π, $w = w_1(M)$ *and*".

page 217, line -1 and page 218, line -8: this is not quite correct; the action on π_3 is given by w if and only if $\pi/F \cong Z$.

page 218, line -6: "(4) follows" should be "(3) and (4) follow".

page 219, line 7: "H^3" should be "H^4".

page 225, line 6: "T_1" should be T_1^*".

page 225, line 11: "M" should be "N".

page 227, line 22: the sentence re the Farrell fibration theorem should be deleted, as it is not clear whether the obstruction in $Wh(\pi)$ is 0.

page 227, line -8: "[Wl78]" should be "[Wl78])".

page 228, line 21: delete "doubly covered by forms".

Chapter 12:

In §12.4 the case when $\pi = Z/4Z$ was overlooked. It is easy to see that in this case $H^*(M; \mathbb{F}_2) \cong \mathbb{F}_2[w, x, y]/(w^2, wx, x^2 + wy, xy, x^3, y^2)$, where w, x and y have degree 1, 2 and 3 respectively; hence $Sq^1x = 0$ and $Sq^1y = wy$.

In §12.7 the structure sets $S_{TOP}(M)$ are treated as groups. While this can be justified, all that is needed here is the cardinality of $S_{TOP}(M)$, which can be derived from the surgery exact sequence of §6.2. Thus on line 9 of page 245 "$S_{TOP}(M) \cong (Z/2Z)^2$" should be "$S_{TOP}(M)$ has four elements", etc.

Chapter 13:

Theorem 13.3 (7): assume also that $\zeta\pi_1(N) \cong Z$. (We must exclude the orientable $\mathbb{H}^2 \times \mathbb{E}^1$-manifolds with nonorientable base orbifolds).

page 255, line 20: replace "Must...?" by "However E need not be geometric. (There are examples with B and F of genus 2 and $\mathrm{Im}(\theta) \cong D$.)".

page 255, line -1: Yes. This follows from Lemma 3.9 of [GLR04], since the mapping class group of the genus 2 surface modulo the central subgroup generated by the hyperelliptic involution is the braid group on 6 strings.

page 260, line 15: "[Te94]" should be "[Tl94]".

Chapter 14:

page 267, line 14: "amiguity" should be "ambiguity".

page 269, line 21: replace ". The isotopy class of θ" by ", which". (This definition is better suited to later usage in Chapter 16).

page 269, lines -12 to -10: replace the sentence begining on this line by "If the monodromy is represented by a self-homeomorphism of finite order then the closed monodromy has finite order.".

page 269, line -8: for example, $\sigma 3_1$. (If the monodromy has finite order then the order is a multiple of 6, since the induced outer automorphism of $F(2) = \pi_1(F)$ has order 6. A self homeomorphism θ of F of finite order extends by coning to a *based* self homeomorphism $\hat{\theta}$ of $\hat{F}$ of the same order, and so induces an automorphism of $F(2)$ of finite order. But $F(2)$ has no automorphism of order 6 [Me74]*. On the other hand we may use the results of [Hn] and [La] to show the closed monodromy has finite order.)

page 273, line 16: "$(n+1)$- and $(n+2)$-handles" should be "$(n+1)$-, $(n+2)$- and $(n+3)$-handles".

page 274, line 24: the fact that n-knot groups have Wirtinger presentations was shown earlier by Yajima [Yj70]*.

page 277, lines 15 and 20: the results asserted in the sentences beginning "We shall show..." are to be found in Theorems 15.13 and 15.7, respectively.

page 280, line 10: the Rapaport conjecture is proven in [Ko05].

Theorem 14.6: replace the sentence "Since $def(\pi) = 1$ ... Theorem 3.6." by "In particular, the manifold M is not aspherical.".

page 282, line 10: "PD_4-group" should be "PD_4^+-group".

page 289, line -10: "k^2. Applying part (1)" should be "$k^2 + 1$. Applying parts (1) and (2)".

Chapter 15:

Theorem 15.7: part (1) can be improved "*if π' is finitely generated then so is A*", while part (3) should be rephrased "*if $r = 1$, $A \leq \zeta\pi'$ and $e(\pi/A) = \infty$ then either A is finitely generated or $M(K)$ is aspherical*".

page 298, line 9: it suffices to assume π' finitely generated. See the remark re Theorem 4.5 above.

page 298, line -1: the reference "[Ga92]" was omitted from the bibliography. See below. (Here we could instead appeal to Theorem 2.14).

page 301, line 21: $\alpha = 3^k n$ here. See [Hi98']* for further details.

page 302, Theorem 15.13: a further equivalent condition is that "π is elementary amenable and has a nontrivial torsion-free normal subgroup". (See Prop. 3 of [Kr93']*.)

page 302, line -10: "by Theorems 15.2 and 15.3" should be "by Theorems 1.17 and 15.2, and by Theorem 15.3".

page 304, line 3: "3.15" should be "3.16".

Chapter 16:

page 310, lines 1-5. The hypothesis "$pqr - qr - pr - pq = \pm 1$ was overlooked here. In fact there are just two such triples: $(2,3,5)$ and $(2,3,7)$.

page 310, line 10: "poly-Z group" should be "poly-Z 2-knot group".

Theorem 16.4: "$\sqrt{\pi} \neq 1$" should be "$\sqrt{\pi}$ is infinite".

page 318, line 9: "$(r, t-n)$" should be "$(t-n, m+np)$".

Chapter 17:

Corollary 17.6.1: "let" should be "Let".

Theorem 17.9: it suffices that π' be finitely generated. See the remark re Theorem 4.5 above.

Theorem 17.10: R was undefined; it should be an open regular neighbourhood of a good slice disc Δ.

page 334, lines 14-15: If $M(K)$ is Seifert fibred it is geometric, by [Ue90] and Theorem 16.2 above (which implies that the monodromy is finite).

In §17.7 we may add the observation that if k is a nontrivial 1-knot and $r \geq 2$ then $M(\tau_r k)$ is geometric if and only if k is simple, and has a geometric decomposition if and only if k is prime. This follows from the fact that the r-fold cyclic branched cover of (S^3, k) admits an equivariant JSJ decomposition, and has finitely generated π_2 if and only if k is prime.

Chapter 18:

page 337, line -10: "Lemma 15.7" should be "Lemma 16.1".

page 347, line -5: delete "(Note that $S^2(p,q,r) \cong S^2(p,q,p'q'))$."

page 351, line -5: "Lemma 11.5" should be "Lemma 11.4".

Bibliography:

[Bo99] has appeared, in J. Reine u. Angew. Math. 576 (2004), 11-62.

[GMT96] has appeared, in Ann. Math. 157 (2003), 335-431.

[Hi02] has appeared, in L'Ens. Math. 48 (2002), 259-262.

[Hi03] has appeared, in J. Math. Soc. Japan 55 (2003), 365-383.

page 365: [HL96] should be [KL96].

[KK99] has appeared in J.Diff. Geom. 69 (2005), 279-352.

The full account of [OS01] has since appeared in [OS02]*.

Index:

Inoue surface: "245" should be "334".

SA: "of" should be "generated by".

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