CATALAN NUMBERS

CATALAN Eugene (1814-1894) wrote an article about these numbers in 1838, but they had been studied by J. von Segner and L. Euler 100 years earlier. They also occur in the work of the Chinese-Mongolian mathematician AN TU MING (1692-1763)

First bracketing problem

A string of left and right brackets is said to be balanced if each left bracket has a matching right bracket.

How many balanced strings of n left and n right brackets are there?

For $n = 0$, there is clearly only 1 way (of doing nothing!).

For $n = 1$, there is 1 way: ()

For $n = 2$, there are 2 ways: (()) and ()()

For $n = 3$, there are 5 ways: ((())), (()()), (())(), ()(()), ()()()

For $n = 4$, there are 14 ways:

(1) (((())))	(2) (((())))	(3) (((())()))	(4) (((()))())
(5) ((()()))	(6) ((())())	(7) ((()())())	(8) ((())(()))
(9) ((()))()()	(10) ()((()()))	(11) ()(()())	(12) ()(())()
(13) ()()(())	(14) ()()()()		

For $n = 5$, there are 42 ways:

(1) ((((()))))	(2) ((((()))))	(3) ((((()))()))	(4) ((((()))()))
(5) (((()))())	(6) (((()(()))))	(7) (((()())))	(8) (((()))())
(9) (((()()))())	(10) (((()))(()))	(11) (((()))()())	(12) (((()))())()
(13) ((()))(())	(14) ((()))()()	(15) (()((())))	(16) (()(()()))
(17) (()(())())	(18) (()((())))()	(19) (()()(()))	(20) (()()()())
(21) (()()())()	(22) (()())(())	(23) (()()))()()	(24) (())((()))
(25) (())(()())	(26) (())(())()	(27) (())()(())	(28) (())()()()
(29) ()(((())))	(30) ()((()()))	(31) ()((())())	(32) ()((()))()
(33) ()(()(()))	(34) ()(()()())	(35) ()(())())()	(36) ()(())(())
(37) ()(())()()	(38) ()()((()))	(39) ()()(()())	(40) ()()(())()
(41) ()()()(())	(42) ()()()()()		

For $n = 6$, there are 132 ways:

(((((()))))), ((((()())))), ((((())()))), ((((()))())), ((((())))()), ((((()))))(), (((()(())))),
(((()()()))), (((()())())), (((()()))()), (((()())))(), (((())(()))), (((())()())), (((())())()),
(((())()))(), (((()))(())), (((()))()()), (((()))())(), (((())))(()), (((())))()(), ((()((())))),
((()(()()))), ((()(())())), ((()(()))()), ((()(())))(), ((()()(()))), ((()()()())), ((()()())()),
((()()()))(), ((()())(())), ((()())()()), ((()())())(), ((()()))(()), ((()()))()(), ((())((()))),
((())(()())), ((())(())()), ((())(()))(), ((())()(())), ((())()()()), ((())()())(), ((())())(()),
((())())()(), ((()))((())), ((()))(()()), ((()))(())(), ((()))()(()), ((()))()()(), (()(((())))),
(()((()()))), (()((())())), (()((()))()), (()((())))(), (()(()(()))), (()(()()())), (()(()())()),
(()(()()))(), (()(())(())), (()(())()()), (()(())())(), (()(()))(()), (()(()))()(), (()()((()))),
(()()(()())), (()()(())()), (()()(()))(), (()()()(())), (()()()()()). (()()()())(), (()()())(()),
(()()())()(), (()())((())), (()())(()()), (()())(())(), (()())()(()), (()())()()(), (())(((()))),
(())((()())), (())((())()), (())((()))(), (())(()(())), (())(()()()), (())(()())(), (())(())(()),
(())(())()(), (())()((())), (())()(()()), (())()(())(), (())()()(()), (())()()()(), ()((((())))),
()(((()()))), ()(((())())), ()(((()))()), ()(((())))(), ()((()(()))), ()((()()())), ()((()())()),
()((()()))(), ()((())(())), ()((())()()), ()((())())(), ()((()))(()). ()((()))()(), ()(()((()))),
()(()(()())), ()(()(())()), ()(()(()))(), ()(()()(())), ()(()()()()), ()(()()())(), ()(()())(()),
()(()())()(), ()(())((())), ()(())(()()), ()(())(())(), ()(())()(()), ()(())()()(), ()()(((()))),
()()((()())), ()()((())()), ()()((()))(), ()()(()(())), ()()(()()()), ()()(()())(), ()()(())(()),
()()(())()(), ()()()((())), ()()()(()()), ()()()(())(), ()()()()(()), ()()()()()().

Balancing Strings of L and R

A string of letters L and R in which every L has a matching R is called balanced.

How many balanced strings of n letters L and n letters R are there?

For $n = 0$, there is clearly only 1 way (of doing nothing!).

For $n = 1$, there is 1 way: LR

For $n = 2$, there are 2 ways: LLRR and LRLR

For $n = 3$, there are 5 ways: LLLRRR, LLRLRR, LLRRLR, LRLLRR, LRLRLR

For $n = 4$, there are 14 ways:

(1) LLLLRRRR (2) LLLRLRRR (3) LLLRRLRR (4) LLLRRRLR
(5) LLRLLRRR (6) LLRLRLRR (7) LLRLRRLR (8) LLRRLLRR
(9) LLRRLRLR (10) LRLLLRRR (11) LRLLRLRR (12) LRLLRRLR
(13) LRLRLLRR (14) LRLRLRLR

For $n = 5$, there are 42 ways:

(1) LLLLLRRRRR (2) LLLLRLRRRR (3) LLLLRRLRRR
(4) LLLLRRRLRR (5) LLLLRRRRLR (6) LLLRLLRRRR
(7) LLLRLRLRRR (8) LLLRLRRLRR (9) LLLRLRRRLR
(10) LLLRRLLRRR (11) LLLRRLRLRR (12) LLLRRLRRLR
(13) LLLRRRLLRR (14) LLLRRRLRLR (15) LLRLLLRRRR
(16) LLRLLRLRRR (17) LLRLLRRLRR (18) LLRLLRRRLR
(19) LLRLRLLRRR (20) LLRLRLRLRR (21) LLRLRLRRLR
(22) LLRLRRLLRR (23) LLRLRRLRLR (24) LLRRLLLRRR
(25) LLRRLLRLRR (26) LLRRLLRRLR (27) LLRRLRLLRR
(28) LLRRLRLRLR (29) LRLLLLRRRR (30) LRLLLRLRRR
(31) LRLLLRRLRR (32) LRLLLRRRLR (33) LRLLRLLRRR
(34) LRLLRLRLRR (35) LRLLRLRRLR (36) LRLLRRLLRR
(37) LRLLRRLRLR (38) LRLRLLLRRR (39) LRLRLLRLRR
(40) LRLRLLRRLR (41) LRLRLRLLRR (42) LRLRLRLRLR

For $n = 6$, there are 132 ways:

LLLLLLRRRRRR, LLLLLRLRRRRR, LLLLLRRLRRRR, LLLLLRRRLRRR,
LLLLLRRRRLRR, LLLLLRRRRRLR, LLLLRLLRRRRR, LLLLRLRLRRRR,
LLLLRLRRLRRR, LLLLRLRRRLRR, LLLLRLRRRRLR, LLLLRRLLRRRR,
LLLLRRLRLRRR, LLLLRRLRRLRR, LLLLRRLRRRLR, LLLLRRRLLRRR,
LLLLRRRRLRLR, LLLLRRRRLRRLR, LLLLRRRRLLRR, LLLLRRRRLRLR,
LLLRLLLRRRRR, LLLRLLRLRRRR, LLLRLLRRLRRR, LLLRLLRRRLRR,
LLLRLLRRRRLR, LLLRLRLLRRRR, LLLRLRLRLRRR, LLLRLRLRRLRR,
LLLRLRLRRRLR, LLLRLRRLLRRR, LLLRLRRLRLRR, LLLRLRRLRRLR,
LLLRLRRRLLRR, LLLRLRRRLRLR, LLLRRLLLRRRR, LLLRRLLRLRRR,
LLLRRLLRRLRR, LLLRRLLRRRLR, LLLRRLRLLRRR, LLLRRLRLRLRR,
LLLRRLRLRRLR, LLLRRLRRLLRR, LLLRRLRRLRLR, LLLRRRLLLRRR,
LLLRRRLLRLRR, LLLRRRLLRRLR, LLLRRRLRLLRR, LLLRRRLRLRLR,
LLRLLLLRRRRR, LLRLLLRLRRRR, LLRLLLRRLRRR, LLRLLLRRRLRR,
LLRLLLRRRRLR, LLRLLRLLRRRR, LLRLLRLRLRRR, LLRLLRLRRLRR,
LLRLLRLRRRLR, LLRLLRRLLRRR, LLRLLRRLRLRR, LLRLLRRLRRLR,
LLRLLRRRLLRR, LLRLLRRRLRLR, LLRLRLLLRRRR, LLRLRLLRLRRR,
LLRLRLLRRLRR, LLRLRLLRRRLR, LLRLRLRLLRRR, LLRLRLRLRLRR,
LLRLRLRLRRLR, LLRLRLRRLLRR, LLRLRLRRLRLR, LLRLRRLLLRRR,
LLRLRRLLRLRR, LLRLRRLLRRLR, LLRLRRLRLLRR, LLRLRRLRLRLR,
LLRRLLLLRRRR, LLRRLLLRLRRR, LLRRLLLRRLRR, LLRRLLLRRRLR,
LLRRLLRLLRRR, LLRRLLRLRLRR, LLRRLLRLRRLR, LLRRLLRRLLRR,
LLRRLLRRLRLR, LLRRLRLLLRRR, LLRRLRLLRLRR, LLRRLRLLRRLR,
LLRRLRLRLLRR, LLRRLRLRLRLR, LRLLLLLRRRRR, LRLLLLRLRRRR,
LRLLLLRRLRRR, LRLLLLRRRLRR, LRLLLLRRRRLR, LRLLLRLLRRRR,
LRLLLRLRLRRR, LRLLLRLRRLRR, LRLLLRLRRRLR, LRLLLRRLLRRR,
LRLLLRRLRLRR, LRLLLRRLRRLR, LRLLLRRRLLRR. LRLLLRRRLRLR,
LRLLRLLLRRRR, LRLLRLLRLRRR, LRLLRLLRRLRR, LRLLRLLRRRLR,
LRLLRLRLLRRR, LRLLRLRLRLRR, LRLLRLRLRRLR, LRLLRLRRLLRR,
LRLLRLRRLRLR, LRLLRRLLLRRR, LRLLRRLLRLRR, LRLLRRLLRRLR,
LRLLRRLRLLRR, LRLLRRLRLRLR, LRLRLLLLRRRR, LRLRLLLRLRRR,
LRLRLLLRRLRR, LRLRLLLRRRLR, LRLRLLRLLRRR, LRLRLRLLLRRR,
LRLRLRLLRLRR, LRLRLRLLRRLR, LRLRLRLRLLRR, LRLRLRLRLRLR .

Connecting with the first bracketing problem

Given a balanced string of left and right brackets. To obtain a balanced string of letters L and R, we replace left bracket (by L and right bracket) by R.

Given a balanced string of letters L and R. To obtain a balanced string of left and right brackets, we replace L by left bracket (and R by right bracket).

1. For each of the following balanced strings of letters L and R, write down the corresponding balanced strings of brackets.

(i) LLLRLRRLRR

(ii) LLRRLLRLRRLR

(iii) LLLRRRLRLLRR

Solution.

The corresponding balanced strings of brackets are:

(i) (((()())()))

(ii) (())(()())()

(iii) ((()))()(()).

2. For each of the following balanced strings of brackets, write down the corresponding balanced strings of letters L and R.

(i) (())()(()())

(ii) (()())(())()()

(iii) (()()())()(())(()())

Solution.

The corresponding balanced strings of letters L and R are

(i) LLRRLRLLRLRR

(ii) LLRLRRLLRRLRLR

(iii) LLRLRLRRLRLLRRLLRLRR

First bracketing problem for $n = 7$

(1) LLLLLLLRRRRRRR

(1) LLLLLLRLRRRRRR
(2) LLLLLLRRLRRRRR
(3) LLLLLLRRRLRRRR
(4) LLLLLLRRRRLRRR
(5) LLLLLLRRRRRLRR
(6) LLLLLLRRRRRRLR

(1) LLLLLRLLRRRRRR
(2) LLLLLRLRLRRRRR
(3) LLLLLRLRRLRRRR
(4) LLLLLRLRRRLRRR
(5) LLLLLRLRRRRLRR
(6) LLLLLRLRRRRRLR
(7) LLLLLRRLLRRRRR
(8) LLLLLRRLRLRRRR
(9) LLLLLRRLRRLRRR
(10) LLLLLRRLRRRLRR
(11) LLLLLRRLRRRRLR
(12) LLLLLRRRLLRRRR
(13) LLLLLRRRLRLRRR
(14) LLLLLRRRLRRLRR
(15) LLLLLRRRLRRRLR
(16) LLLLLRRRRLLRRR
(17) LLLLLRRRRLRLRR
(18) LLLLLRRRRLRRLR
(19) LLLLLRRRRRLLRR
(20) LLLLLRRRRRLRLR

(1) LLLLRLLLRRRRRR
(2) LLLLRLLRLRRRRR
(3) LLLLRLLRRLRRRR
(4) LLLLRLLRRRLRRR
(5) LLLLRLLRRRRLRR
(6) LLLLRLLRRRRRLR
(7) LLLLRLRLLRRRRR
(8) LLLLRLRLRLRRRR
(9) LLLLRLRLRRLRRR
(10) LLLLRLRLRRRLRR
(11) LLLLRLRLRRRRLR
(12) LLLLRLRRLLRRRR
(13) LLLLRLRRLRLRRR
(14) LLLLRLRRLRRLRR
(15) LLLLRLRRLRRRLR
(16) LLLLRLRRRLLRRR
(17) LLLLRLRRRLRLRR
(18) LLLLRLRRRLRRLR
(19) LLLLRLRRRRLLRR
(20) LLLLRLRRRRLRLR
(21) LLLLRRLLLRRRRR
(22) LLLLRRLLRLRRRR
(23) LLLLRRLLRRLRRR
(24) LLLLRRLLRRRLRR
(25) LLLLRRLLRRRRLR
(26) LLLLRRLRLLRRRR
(27) LLLLRRLRLRLRRR
(28) LLLLRRLRLRRLRR
(29) LLLLRRLRLRRRLR
(30) LLLLRRLRRLLRRR
(31) LLLLRRLRRLRLRR
(32) LLLLRRLRRLRRLR
(33) LLLLRRLRRRLLRR
(34) LLLLRRLRRRLRLR
(35) LLLLRRRLLLRRRR
(36) LLLLRRRLLLRLRR
(37) LLLLRRRLLRRLRR
(38) LLLLRRRLLRRRLR
(39) LLLLRRRLRLLRRR
(40) LLLLRRRLRLRLRR
(41) LLLLRRRLRLRRLR
(42) LLLLRRRLRRLLRR

(43) LLLLRRRLRRLRLR (44) LLLLRRRRLLLRRR
(45) LLLLRRRRLLRLRR (46) LLLLRRRRLLRRLR
(47) LLLLRRRRLRLLRR (48) LLLLRRRRLRLRLR

(1) LLLRLLLLRRRRRR (2) LLLRLLLLRLRRRR
(3) LLLRLLLRRLRRRR (4) LLLRLLLRRRLRRR
(5) LLLRLLLRRRRLRR (6) LLLRLLLRRRRRLR
(7) LLLRLLRLLRRRRR (8) LLLRLLRLRLRRRR
(9) LLLRLLRLRRLRRR (10) LLLRLLRLRRRLRR
(11) LLLRLLRLRRRRLR (12) LLLRLLRRLLRRRR
(13) LLLRLLRRLRLRRR (14) LLLRLLRRLRRLRR
(15) LLLRLLRRLRRRLR (16) LLLRLLRRRLLRRR
(17) LLLRLLRRRLRLRR (18) LLLRLLRRRLRRLR
(19) LLLRLLRRRRLLRR (20) LLLRLLRRRRLRLR
(21) LLLRLRLLLRRRRR (22) LLLRLRLLRLRRRR
(23) LLLRLRLLRRLRRR (24) LLLRLRLLRRRLRR
(25) LLLRLRLLRRRRLR (26) LLLRLRLRLLRRRR
(27) LLLRLRLRLRLRRR (28) LLLRLRLRLRRLRR
(29) LLLRLRLRLRRRLR (30) LLLRLRLRRLLRRR
(31) LLLRLRLRRLRLRR (32) LLLRLRLRRLRRLR
(33) LLLRLRLRRRLLRR (34) LLLRLRLRRRLRLR
(35) LLLRLRRLLLRRRR (36) LLLRLRRLLRLRRR
(37) LLLRLRRLLRRLRR (38) LLLRLRRLLRRRLR
(39) LLLRLRRLRLLRRR (40) LLLRLRRLRLRLRR
(41) LLLRLRRLRLRRLR (42) LLLRLRRLRRLLRR
(43) LLLRLRRLRRLRLR (44) LLLRLRRRLLLRRR
(45) LLLRLRRRLLRLRR (46) LLLRLRRRLLRRLR
(47) LLLRLRRRLRLLRR (48) LLLRLRRRLRLRLR
(49) LLLRRLLLLRRRRR (50) LLLRRLLLRLRRRR
(51) LLLRRLLLRRLRRR (52) LLLRRLLLRRRLRR
(53) LLLRRLLLRRRRLR (54) LLLRRLLRLLRRRR
(55) LLLRRLLRLRLRRR (56) LLLRRLLRLRRLRR
(57) LLLRRLLRLRRRLR (58) LLLRRLLRRLLRRR
(59) LLLRRLLRRLRLRR (60) LLLRRLLRRLRRLR
(61) LLLRRLLRRRLLRR (62) LLLRRLLRRRLRLR
(63) LLLRRLRLLLRRRR (64) LLLRRLRLLRLRRR
(65) LLLRRLRLLRRLRR (66) LLLRRLRLLRRRLR
(67) LLLRRLRLRLLRRR (68) LLLRRLRLRLRLRR
(69) LLLRRLRLRLRRLR (70) LLLRRLRLRRLLRR

(71) LLLRRLRLRRLRLR (72) LLLRRLRRLLLRRR
(73) LLLRRLRRLLRLRR (74) LLLRRLRRLLRRLR
(75) LLLRRLRRLRLLRR (76) LLLRRLRRLRLRLR
(77) LLLRRRLLLLRRRR (78) LLLRRRLLLRLRRR
(79) LLLRRRLLLRRLRR (80) LLLRRRLLLRRRLR
(81) LLLRRRLLRLLRRR (82) LLLRRRLLRLRLRR
(83) LLLRRRLLRLRRLR (84) LLLRRRLLRRLLRR
(85) LLLRRRLLRRLRLR (86) LLLRRRLRLLLRRR
(87) LLLRRRLRLLRLRR (88) LLLRRRLRLLRRLR
(89) LLLRRRLRLRLLRR (90) LLLRRRLRLRLRLR

(1) LLRLLLLLRRRRRR (2) LLRLLLLRLRRRRR
(3) LLRLLLLRRLRRRR (4) LLRLLLLRRRLRRR
(5) LLRLLLLRRRRLRR (6) LLRLLLLRRRRRLR
(7) LLRLLLRLLRRRRR (8) LLRLLLRLRLRRRR
(9) LLRLLLRLRRLRRR (10) LLRLLLRLRRRLRR
(11) LLRLLLRLRRRRLR (12) LLRLLLRRLLRRRR
(13) LLRLLLRRLRLRRR (14) LLRLLLRRLRRLRR
(15) LLRLLLRRLRRRLR (16) LLRLLLRRRLLRRR
(17) LLRLLLRRRLRLRR (18) LLRLLLRRRLRRLR
(19) LLRLLLRRRRLLRR (20) LLRLLLRRRRLRLR
(21) LLRLLRLLLRRRRR (22) LLRLLRLLRLRRRR
(23) LLRLLRLLRRLRRR (24) LLRLLRLLRRRLRR
(25) LLRLLRLLRRRRLR (26) LLRLLRLRLLRRRR
(27) LLRLLRLRLRLRRR (28) LLRLLRLRLRRLRR
(29) LLRLLRLRLRRRLR (30) LLRLLRLRRLLRRR
(31) LLRLLRLRRLRLRR (32) LLRLLRLRRLRRLR
(33) LLRLLRLRRRLLRR (34) LLRLLRLRRRLRLR
(35) LLRLLRRLLLRRRR (36) LLRLLRRLLRLRRR
(37) LLRLLRRLLRRLRR (38) LLRLLRRLLRRRLR
(39) LLRLLRRLRLLRRR (40) LLRLLRRLRLRLRR
(41) LLRLLRRLRLRRLR (42) LLRLLRRLRRLLRR
(43) LLRLLRRLRRLRLR (44) LLRLLRRRLLLRRR
(45) LLRLLRRRLLRLRR (46) LLRLLRRRLLRRLR
(47) LLRLLRRRLRLLRR (48) LLRLLRRRLRLRLR
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(51) LLRLRLLLRRLRRR (52) LLRLRLLLRRRLRR
(53) LLRLRLLLRRRRLR (54) LLRLRLLRLLRRRR
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(57) LLRLRLLRLRRRLR
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(59) LLRLRLLRRLRLRR
(60) LLRLRLLRRLRRLR
(61) LLRLRLLRRRLLRR
(62) LLRLRLLRRRLRLR
(63) LLRLRLRLLLRRRR
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(65) LLRLRLRLLRRLRR
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(68) LLRLRLRLRLRLRR
(69) LLRLRLRLRLRRLR
(70) LLRLRLRLRRLLRR
(71) LLRLRLRLRRLRLR
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(75) LLRLRLRRLRLLRR
(76) LLRLRLRRLRLRLR
(77) LLRLRRLLLLRRRR
(78) LLRLRRLLLRLRRR
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(126) LLRRLRLLRRLLRR
(127) LLRRLRLLRRLRLR
(128) LLRRLRLRLLLRRR
(129) LLRRLRLRLLLRLR
(130) LLRRLRLRLLRRLR
(131) LLRRLRLRLRLLRR
(132) LLRRLRLRLRLRLR

(1) LRLLLLLLRRRRRR
(2) LRLLLLLRLRRRRR
(3) LRLLLLLRRLRRRR
(4) LRLLLLLRRRLRRR
(5) LRLLLLLRRRRLRR
(6) LRLLLLLRRRRRLR
(7) LRLLLLRLLRRRRR
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(14) LRLLLLRRLRRLRR
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(40) LRLLLRRLRLRLRR
(41) LRLLLRRLRLRRLR
(42) LRLLLRRLRRLLRR
(43) LRLLLRRLRRLRLR
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(50) LRLLRLLLLRLRRR
(51) LRLLRLLLLRRLRR
(52) LRLLRLLLLRRRLR
(53) LRLLRLLLLRRRRLR
(54) LRLLRLLRLLRRRR
(55) LRLLRLLRLRLRRR
(56) LRLLRLLRLRRLRR
(57) LRLLRLLRLRRRLR
(58) LRLLRLLRRLLRRR
(59) LRLLRLLRRLRLRR
(60) LRLLRLLRRLRRLR
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(62) LRLLRLLRRRLRLR
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