Stacking of Dominos Problem

Problem: For any integer \( n > 0 \), we are forming a stack of dominos with the first domino at \( (0, 0) \), each domino must either sit directly on one below or else overlap by half its length and the stack must be contained in the first quadrant.

For any \( n \), how many such stacks are there?

For \( n = 1 \), there is clearly only 1 such stack; __________

For \( n = 2 \), there are 2 such stacks: __________

For \( n = 3 \), there are 5 such stacks:

__________ __________

For \( n = 4 \), there are 14 such stacks:

__________ __________ __________ __________

__________ __________ __________ __________

__________ __________ __________ __________

For \( n = 5 \), there are 42 such stacks:

__________ __________ __________ __________

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In fact, for any integer $n \geq 0$, the number of such stackings is the Catalan number $c_n$.

**Connection with the first bracket problem**

Given a balanced string of left and right brackets, we obtain the stacking of dominos as follows: Each left bracket gives rise to a domino. Draw the first domino corresponding to the first left bracket. If $L$ is the next bracket, then the second domino is obtained by moving to the right by half of its length to sit over the first domino. If the $L$ was followed by $R$, then $L$, then the next domino is obtained by moving to the left by half of its length and then right by half of its length and so the domino will sit over the previous domino. If $L$ was followed by $k$ $R$’s and then $L$, then the next domino is obtained by moving the left by $k$ times half of its length and then right by half of its length and so the domino will sit over the previous $k - 1$ domino. Continue the same procedure till all the $L$’s have been used up.

Given a stacking of dominoes, we obtain a balanced string of left and right brackets by reversing the above procedures.

1. Construct the stacking of dominos corresponding to the following balanced strings of brackets:
   
   (i) LLRLRLRLRRR
   (ii) LLRLRLRLLRRLR
   (iii) LLRLRLRLLRRLRLR

   **Solution.**

   The corresponding ways of stacking the dominos are:

   (i) ![Image](image1.png)
   (ii) ![Image](image2.png)
   (iii) ![Image](image3.png)
2. Construct balanced strings of brackets corresponding to the following stacking of dominoes:

(i)  

(ii)  

(iii)  

(iv)  

Solution.

The corresponding balanced strings of brackets are:

(i) LLLRRLRLLRLLRRRRRLLLRR

(ii) LLLRRLRLLRLLRLLRRRRRRRLLLRR

(iii) LLLRRRRRLLLLRLLLLRLLLLRRRRRR

(iv) LLLRRRRRLLLLRLLLLRLLLLRRRRRR