**Mountain Ranges**

*Problem 1:* How many mountain ranges can you draw with $n$ upstrokes and $n$ downstrokes?

*Problem 2:* A Dyck path from $(0, 0)$ to $(2n, 0)$ is lattice path with steps $(1,1)$ and $(1,-1)$, never falling below the $x$-axis. How many such Dyck paths?

*Problem 3:* Consider a random walk in the plane, where the steps are from $(x, y)$ to $(x+1, y+1)$ or $(x+1, y-1)$, starting at a given point. In how many ways can the random walk go from $(0,0)$ to $(2n,0)$ through the upper half plane without crossing the $x$-axis?

For $n = 1$, there is only 1 such mountain range and for $n = 2$, there are 2 such mountain ranges.

For $n = 3$, there are 5 such mountain ranges.

For $n = 4$ there are 14 such mountain ranges:

For $n = 5$ there are 42 such mountain ranges:
In fact, the number of mountain ranges with \( n \) upstrokes and \( n \) downstrokes is the Catalan number \( c_n \).

**Connection with the first bracketing problem**

Given a balanced string of brackets, we obtain a mountain ranges as follows. Replace the left brackets by upstrokes and right brackets by downstrokes.

Given a mountain ranges of upstrokes and downstrokes, we obtain a balanced string of brackets as follows. Replace upstrokes by left brackets and downstrokes by right brackets.

1. Construct balanced strings of brackets corresponding to the following mountain ranges.

![Mountain ranges](image)

\( (i) \) \hspace{1cm} \( (ii) \) \hspace{1cm} \( (iii) \)

**Solution.**

The corresponding balanced strings of brackets are:

\( (i) \) \((\text{()()})\)
\( (ii) \) \((\text{()()})\)
\( (iii) \) \((\text{()()})\)

2. For each of the following balanced strings of brackets, construct the corresponding mountain ranges.

\( (i) \) \((\text{()()})\)
\( (ii) \) \((\text{()()})\)
\( (iii) \) \((\text{()()})\)

**Solution.**

The corresponding mountain ranges are:

![Mountain ranges](image)

\( (i) \) \hspace{1cm} \( (ii) \) \hspace{1cm} \( (iii) \)