

### Mountain Ranges

*Problem 1:* How many mountain ranges can you draw with  $n$  upstrokes and  $n$  downstrokes?

*Problem 2:* A Dyck path from  $(0,0)$  to  $(2n,0)$  is lattice path with steps  $(1,1)$  and  $(1,-1)$ , never falling below the  $x$ -axis. How many such Dyck paths?

*Problem 3:* Consider a random walk in the plane, where the steps are from  $(x,y)$  to  $(x+1,y+1)$  or  $(x+1,y-1)$ , starting at a given point. In how many ways can the random walk go from  $(0,0)$  to  $(2n,0)$  through the upper halfplane without crossing the  $x$ -axis?

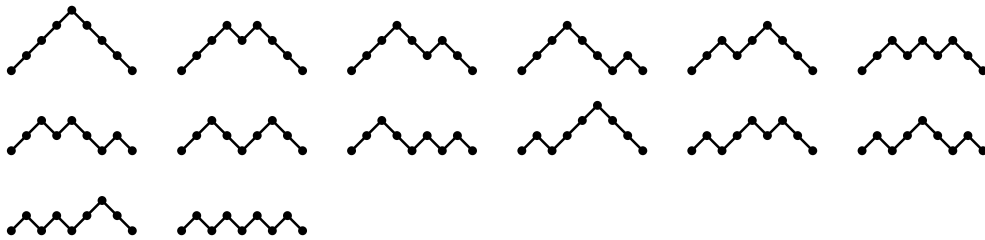
For  $n = 1$ , there is only 1 such mountain range and for  $n = 2$ , there are 2 such mountain ranges.



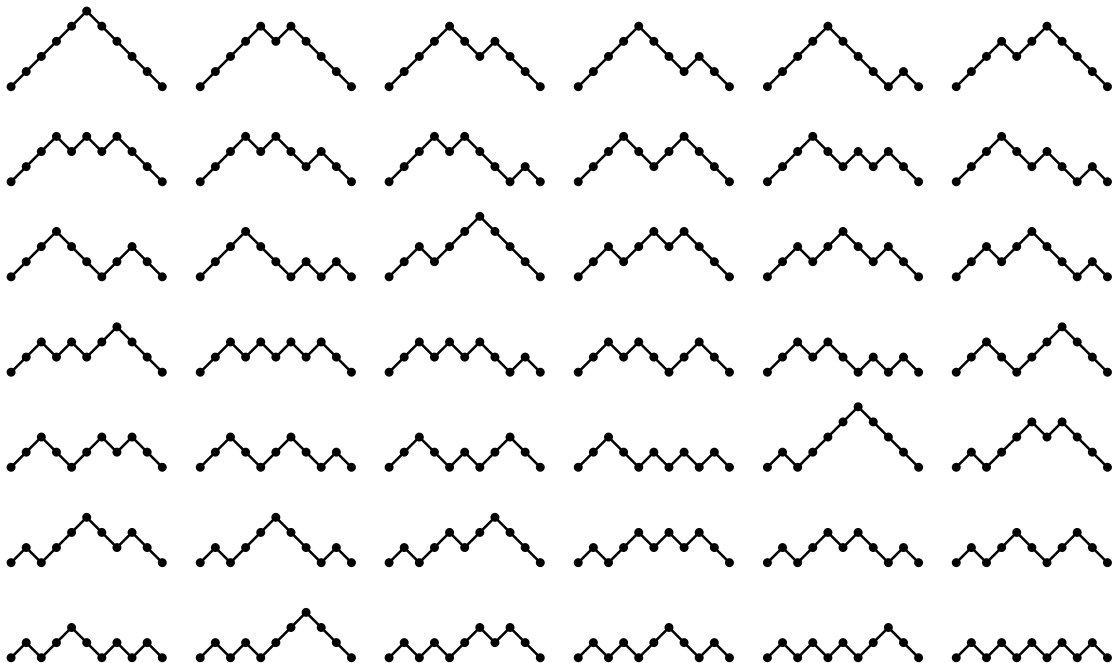
For  $n = 3$ , there are 5 such mountain ranges.



For  $n = 4$  there are 14 such mountain ranges:



For  $n = 5$  there are 42 such mountain ranges:



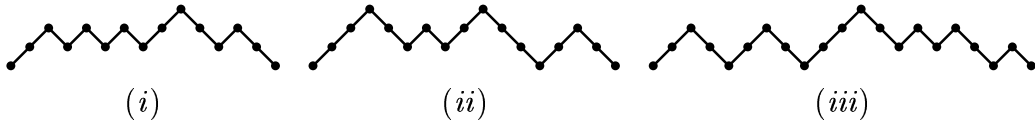
In fact, the number of mountain ranges with  $n$  upstrokes and  $n$  downstrokes is the Catalan number  $c_n$ .

*Connection with the first bracketing problem*

Given a balanced string of brackets, we obtain a mountain ranges as follows. Replace the left brackets by upstrokes and right brackets by downstrokes.

Given a mountain ranges of upstrokes and downstrokes, we obtain a balanced string of brackets as follows. Replace upstrokes by left brackets and downstrokes by right brackets.

1. Construct balanced strings of brackets corresponding to the following mountain ranges.



*Solution.*

The corresponding balanced strings of brackets are:

- (i)  $((()()())())$   
(ii)  $((())()())()$   
(iii)  $((())()((())()())())$

2. For each of the following balanced strings of brackets, construct the corresponding mountain ranges.

- (i)  $((())()())$   
(ii)  $((())()())()$   
(iii)  $((())()()())()$

*Solution.*

The corresponding mountain ranges are:

