**Partitions**

**Problem:** How many partitions \( \{B_1, B_2, \ldots, B_n\} \) of \( \{1, 2, \ldots, n\} \) are there such that if the numbers 1, 2, \ldots, \( n \) are arranged in order around a circle, then the convex hulls of the blocks \( B_1, B_2, \ldots, B_n \) are pairwise disjoint?

For \( n = 1 \), there is only 1 way; namely

For \( n = 2 \), there are 2 ways; namely \( \cdot \cdot \) and \( \cdot \cdot \).

For \( n = 3 \), there are 5 ways:

\[
\begin{array}{cccc}
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

For \( n = 4 \), there are 14 ways:

\[
\begin{array}{cccc}
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

For \( n = 5 \) there are 42 ways:

\[
\begin{array}{cccc}
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\end{array}
\]
For \( n = 6 \), there are 132 such partitions:
In fact, for any $n$, the number of such partitions is the Catalan number $c_n$. 
Connection with the first bracket problem

Given a balanced string of $n$ left and $n$ right brackets, we obtain the corresponding partition as follows. First we choose a starting position, choose clockwise direction to draw $n$ dots and name these dots from 1 to $n$. Also name the left brackets in the balanced string from 1 to $n$. Read from the left of the balanced string: if there is a block of $k$ right brackets $R$, then join the integers corresponding to the matching $L$ by a $k$–gon [1-gon is a point, 2-gon is a line].

Given such a partition, we first choose a starting position for the partition and choose clockwise direction to construct the corresponding balanced string of bracket, by reversing the above procedures.

Remark: This is closely related to the noncrossing partitions problem and the non-crossing Murasaki diagrams problem.

1. Construct balanced strings of brackets corresponding to the following partition:

   (i) \[ \text{1 2 3} \quad \text{4} \quad \text{5 6} \quad \text{7} \]
   (ii) \[ \text{1 2 3 4} \quad \text{5} \quad \text{6 7 8} \quad \text{9 10} \]
   (iii) \[ \text{1 2 3 4} \quad \text{5} \quad \text{6 7 8 9} \quad \text{10 11 12} \]

   Solution. The corresponding balanced strings of brackets are:
   (i) \[ \text{LLLRLRLLRLLLRR} \]
   (ii) \[ \text{LRLRLRLRRLRLLLLL} \]
   (iii) \[ \text{LLLRLRLLRLLRLLLRR} \]

2. For the following balanced strings of brackets, construct the corresponding partitions:

   (i) \[ \text{LLLRLLLLRLRRR} \]
   (ii) \[ \text{LRLLRLRLRLRRR} \]
   (iii) \[ \text{LRLRRLRRRLLLRRRLLRRR} \]

   Solution. The corresponding partitions are:

   (i) \[ \text{1 2 3} \quad \text{4} \quad \text{5 6} \quad \text{7} \]
   (ii) \[ \text{1 2 3 4} \quad \text{5} \quad \text{6 7} \quad \text{8} \]
   (iii) \[ \text{1 2 3 4} \quad \text{5} \quad \text{6 7 8} \quad \text{9 10 11} \quad \text{12} \]