

Murasaki diagrams

Problem 1: Given n vertical lines. A diagram obtained by joining some of the n vertical lines with horizontal lines, is called a *Murasaki diagram*. How many noncrossing Murasaki diagrams with n vertical lines are there?

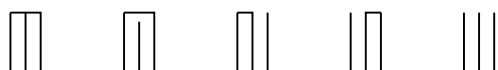
For $n = 1$, there is only 1 such diagram, just a vertical line:



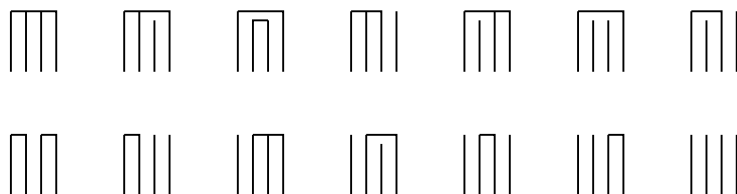
For $n = 2$, there are 2 such diagrams:



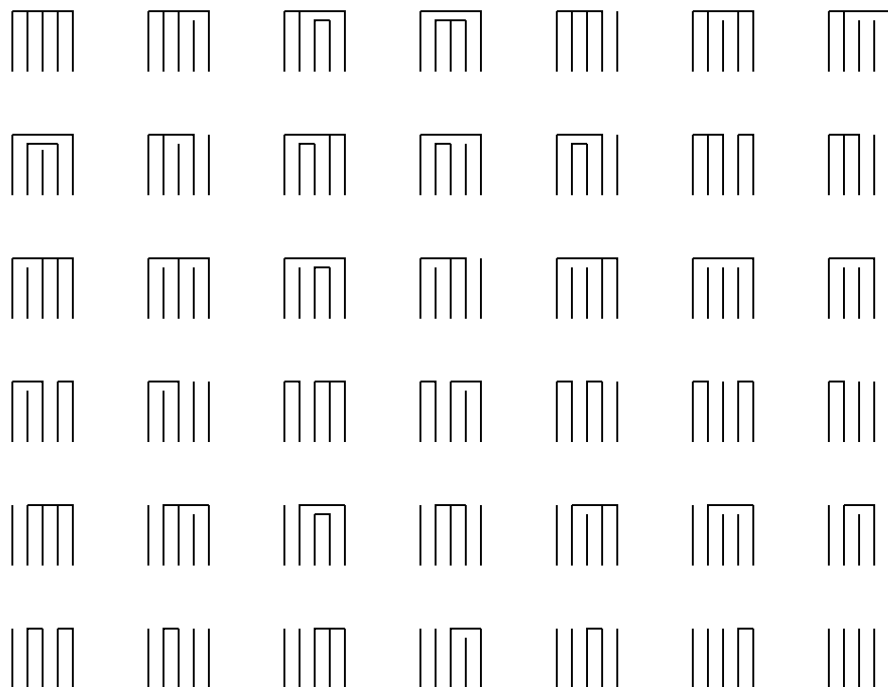
For $n = 3$, there are 5 such diagrams:



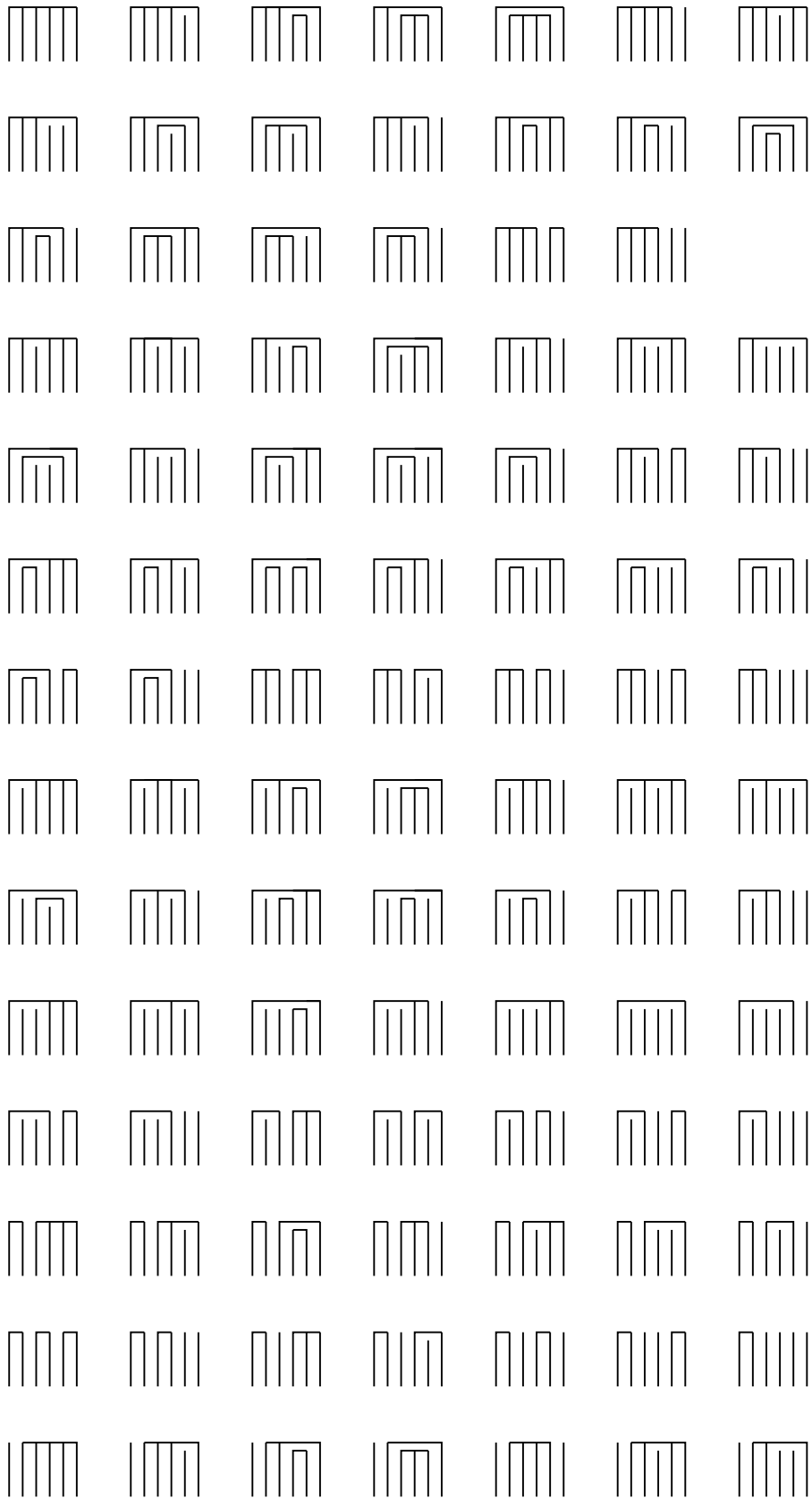
For $n = 4$, there are 14 such diagrams:

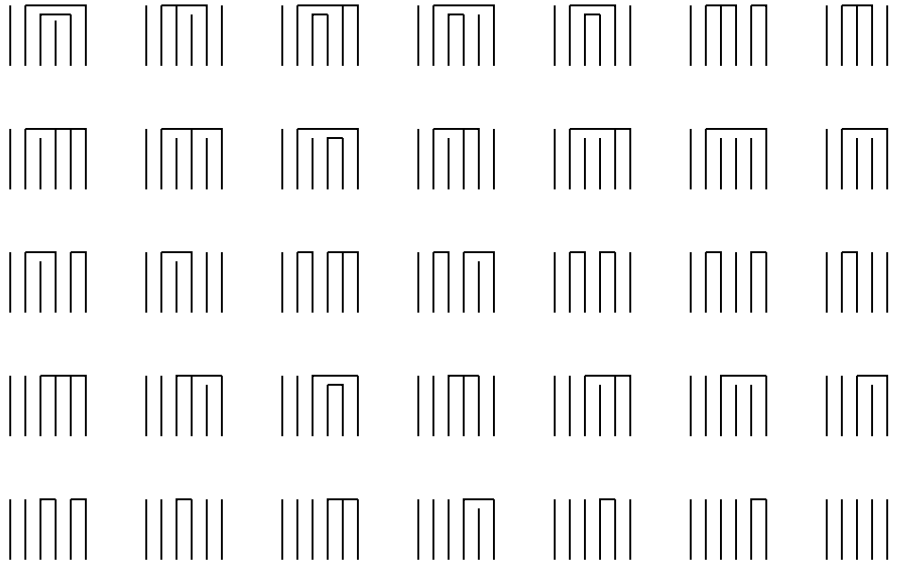


For $n = 5$, there are 42 such diagrams:



For $n = 6$, there are 132 such diagrams:





In fact, for any n , the number of noncrossing Murasaki diagrams is the Catalan number c_n

Connection with the first bracket problem

Given a balanced strings of n left and n right brackets, we obtain the corresponding Murasaki diagram as follows: Draw n vertical lines and label them from 1 to n from the left. Label the left brackets from 1 to n in the given balanced string of brackets. Then each vertical line corresponds to a left bracket. For each group of right brackets together, we obtain the positions of the matching left brackets and then we join the corresponding vertical lines by a horizontal line from the top of the lines.

Given a Murasaki diagram, we obtain a balanced string of left and right brackets as follows: Label the n lines from 1 to n from the left. To each line, we associate a left bracket. For each group of the lines joined by a horizontal line, we associate the corresponding left brackets by the matching right brackets group together.

Remark: The noncrossing Murasaki diagrams correspond exactly to the noncrossing partitions. This is also closely related to the other partitions problem.

