




Arranging circles on the same line

How many ways to draw n nonintersecting circles with centres on the same line are there? Any such diagram is referred to as a string of circles.

For $n = 0$, there is clearly only 1 way (of doing nothing!).

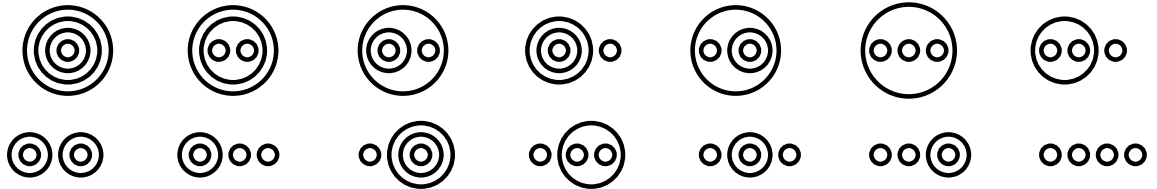
For $n = 1$, there is 1 way: 

For $n = 2$, there are 2 ways:  

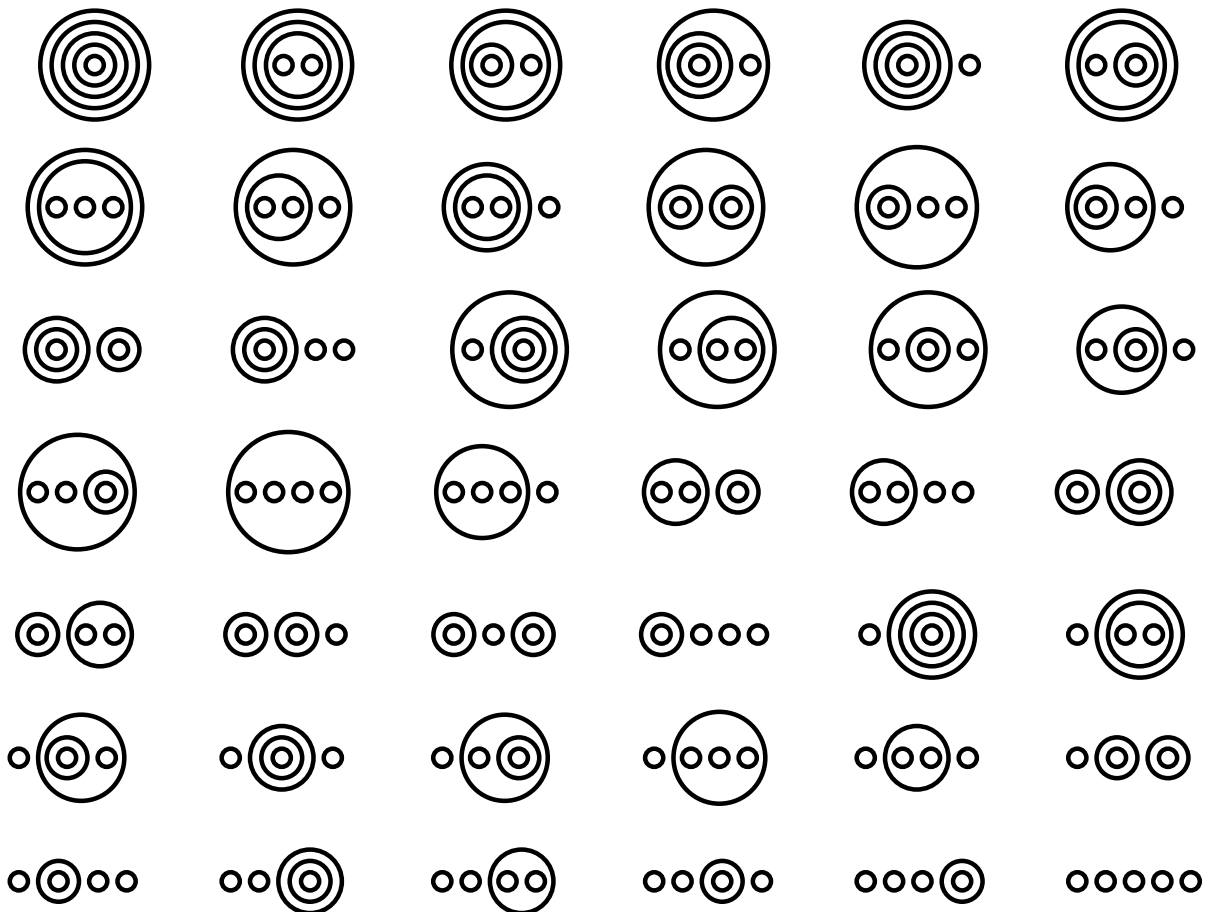
For $n = 3$, there are 5 ways:



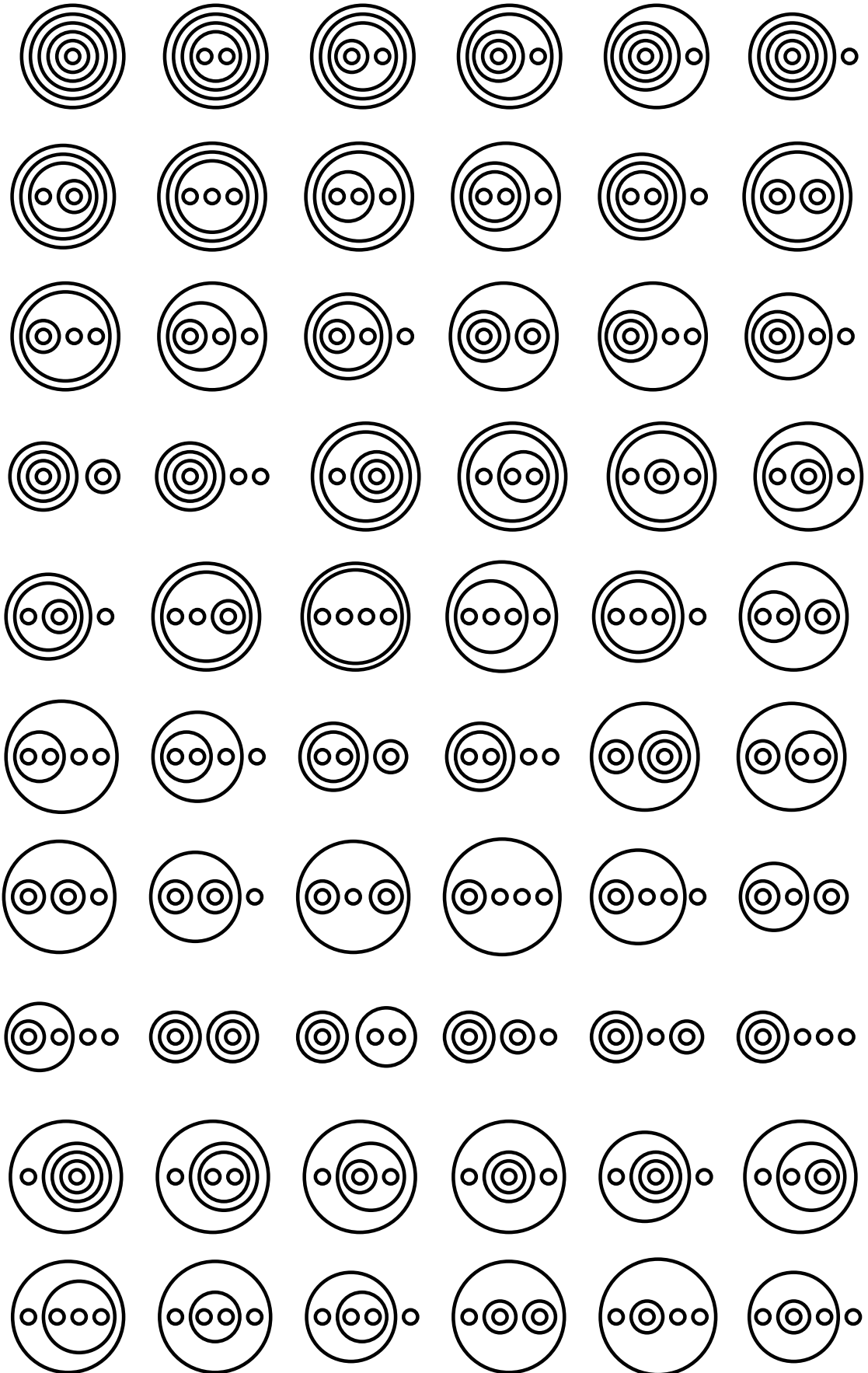
For $n = 4$, there are 14 ways:

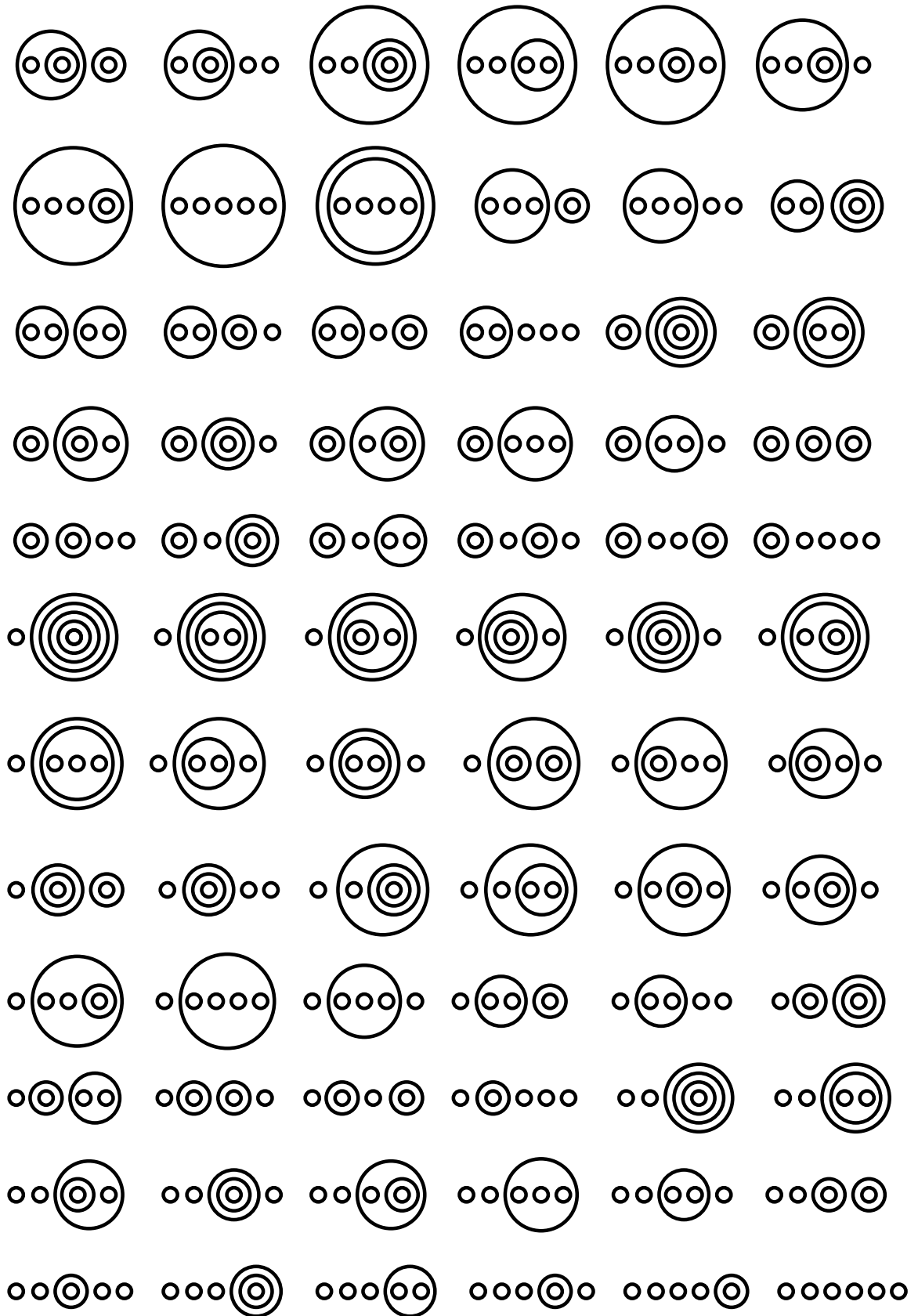


For $n = 5$, there are 42 ways:



For $n = 6$, there are 132 ways:





Connecting with the first bracketing problem

Given a balanced string of left and right brackets, to obtain the corresponding string of circles, we replace a matching pair LR by a circle.

Given a string of circles, to obtain a balanced string of left and right brackets, we replace a circle by a matching pair LR.

1. For each of the following balanced strings of letters L and R, write down the corresponding balanced strings of brackets.

- (i) LLLRLRRLRR
(ii) LLRLLLRRLRLR
(iii) LLLRRRLRLLRR

Solution.

The corresponding balanced strings of circles are:



(i)

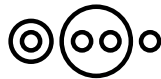


(ii)



(iii)

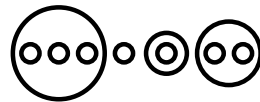
2. For each of the following strings of circles, write down the corresponding balanced strings of brackets.



(i)



(ii)



(iii)

Solution.

The corresponding balanced strings of brackets are

- (i) LLRRLRLLRRLRR
(ii) LLRLRRLRRLRLRLR
(iii) LLRLRRLRRLRLLRLLRRLRR