Stacking of manilla folders

Problem: How many ways to stack \( n \) (identical) manilla folders, upright, nested, into your carrying bag are there?

For \( n = 0 \), there is 1 way.

For \( n = 1 \), there is also 1 way:

For \( n = 2 \), there are 2 way:

For \( n = 3 \), there are 5 way:

For \( n = 4 \), there are 14 way:
For $n = 5$, there are 42 ways:
For $n = 6$, there are 132 ways:
Connecting with the first bracketing problem

Given a balanced string of left and right brackets, to obtain the corresponding string of circles, we replace a matching pair LR by a manilla folder.

Given a stacking of manilla folders, to obtain a balanced string of left and right brackets, we replace a folder by a matching pair LR.

1. For each of the following balanced strings of letters L and R, write down the corresponding stacking of manilla folders.
   
   (i) LLLRLRRLRR
   (ii) LLRRLLRLRRLR
   (iii) LLLRRRLRLLRR

   **Solution.**
   
   The corresponding stacking of manilla folders are:

   
   ![Stacking of manilla folders]

   (i) ![Stack](image1)
   (ii) ![Stack](image2)
   (iii) ![Stack](image3)

2. For each of the following strings of circles, write down the corresponding balanced strings of brackets.

   
   ![Strings of circles]

   (i) ![Circle](image4)
   (ii) ![Circle](image5)
   (iii) ![Circle](image6)

   **Solution.**
   
   The corresponding balanced strings of brackets are

   (i) LLRRLRLRLRRLR
   (ii) LLRLRRLRLRLRRLR
   (iii) LLRLRRLRLRLRRLRRLR