THE UNIVERSITY OF SYDNEY
FRACULTIES OF ARTS, ECONOMICS, EDUCATION,
ENGINEERING AND SCIENCE

MATH1004
DISCRETE MATHEMATICS

November 2001

TIME ALLOWED: One and a half Hours

LECTURER: K-G Choo and J N Ward

This Examination has 4 Printed Components.

(1) AN EXTENDED ANSWER QUESTION PAPER (THIS BOOKLET, WHITE):
0 pages numbered 1 to 0; 0 questions numbered 1 to 0.

(2) A GRAPH SHEET (YELLOW): 1 page.

(3) A MULTIPLE CHOICE QUESTION PAPER (YELLOW):
0 pages numbered 1 to 0; 15 questions numbered 1 to 15.

(4) A MULTIPLE CHOICE ANSWER SHEET (WHITE): 1 page.

Components 2, 3 and 4 MUST NOT be removed from the examination room.

This Examination has 2 Sections: Extended Answer and Multiple Choice.

The Extended Answer Section is worth 70% of the total marks for the paper:
all questions may be attempted; questions are of equal value;
working must be shown;
question 3(ii)(a) must be answered on the Graph Sheet

The Multiple Choice Section is worth 30% of the total marks for the paper:
all questions may be attempted; questions are of equal value;
answers must be coded onto the Multiple Choice Answer Sheet.

Calculators will be supplied; no other electronic calculators are permitted.
1. (i) For the Boolean function

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>z</th>
<th>f(x, y, z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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write down its Boolean expression in disjunctive normal form.

(ii) For the Boolean expression

\[ x(y \lor z') \lor y'z \]

describe the corresponding Boolean function \( g(x, y, z) \) by completing the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>z</th>
<th>g(x, y, z)</th>
</tr>
</thead>
<tbody>
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</table>

(iii) Consider the following Karnaugh map:

\[
\begin{array}{c|c|c|c|c}
wx & & & & \\
wx' & & & & \\
w'x & & & & \\
w'x' & & & & \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
yz & yz' & y'z' & y'z \\
\hline
1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 \\
\end{array}
\]

(a) Write down a corresponding simple Boolean expression.

(b) Draw a switching circuit for the simple Boolean expression in (a).

(c) Draw a digital logic circuit, using AND and OR gates and inverters for the simple Boolean expression in (a).
2. (i) Let $A$, $B$ and $C$ be any three sets. Prove that
\[(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C).\]

(ii) Construct the truth table for the proposition $(\sim p \land q) \Rightarrow (p \lor q)$ and decide whether it is a tautology, a contradiction or neither.

(iii) Let $\mathbb{Z}$ be the set of all integers and $\mathbb{N}$ the set of all natural numbers. Decide whether the following propositions are true or false. Give reasons for your answers.
(a) $(\forall x \in \mathbb{Z})(x^2 = 9) \Rightarrow (x^2 - 2x - 3 = 0))$
(b) $(\exists x \in \mathbb{N})(x^2 = 9) \land (x^2 - 2x - 3 = 0))$

3. (i) How many distinguishable arrangements are there of the letters in the word GOONDOOWINDI?

(ii) How many such distinguishable arrangements are there, with the two N’s together?

(iii) How many such distinguishable arrangements are there, with the two N’s together and with the two D’s together?

(iv) How many such distinguishable arrangements are there, with the two N’s together, with the two D’s together and with the two I’s together?

(v) How many such distinguishable arrangements are there, with the two N’s together, or the two D’s together or the two I’s together? [Use Principle of Inclusion-Exclusion]

(vi) How many such distinguishable arrangements are there, with the two N’s not together, the two D’s not together and the two I’s not together? [Leave all your answers in factorial form and do not simplify them.]

4. (i) Use induction to prove that for all integers $n \geq 1$,
\[1 \times 1! + 2 \times 2! + \ldots + n \times n! = (n + 1)! - 1.\]

(ii) Solve the recurrence relation $x_{n+2} - x_{n+1} - 6x_n = 0$, for all $n \geq 0$, with $x_0 = 3$ and $x_1 = 4$.

5. (i) Let $a_0, a_1, a_2, \ldots$ be the sequence determined by the recurrence relation
\[a_n - 2a_{n-1} = 4^{n-1}\]
for all $n \geq 1$, with $a_0 = 1$. Find a closed form for its generating function and thereby find a formula for $a_n$.

(ii) Let $A$ be a 10-element subset of $\{1, 2, 3, \ldots, 50\}$. Use Pigeonhole Principle to show that $A$ possesses two different 5-element subsets, the sums of whose elements are equal.