The Perron-Frobenius Theorem for Leslie matrices
(age-structured populations)

(Intuitive) definition: A system is said to be *ergodic* if its eventual behaviour is independent of its initial state.

Definition: A *Leslie matrix* $\mathbf{L}$ is a non-negative matrix which has non-zero elements only in the top row and on the first diagonal line of elements immediately below the matrix diagonal. All other elements are always zero.

Theorem For any Leslie matrix $\mathbf{L}$ there exists a real positive eigenvalue λ_1 that is a simple root of the characteristic equation. This eigenvalue, which is called the *dominant eigenvalue*, is strictly greater in magnitude than any other eigenvalue. The associated right eigenvector $\mathbf{w}_1$ and left eigenvector $\mathbf{v}_1$ are both real and the only strictly positive right and left eigenvalues of $\mathbf{L}$.

Corollary: The dominant eigenvalue determines the ergodic properties of the population:
if $\lambda_1 > 1$ then $\mathbf{n}(t) \to A\lambda^t \mathbf{w}_1$;
if $\lambda_1 < 1$ then $\mathbf{n}(t) \to 0$ as $t \to \infty$.

The right eigenvector is proportional to the stable age distribution. It can be rescaled to give either the proportion or the percentage of individuals in each age class.

The left eigenvector is the *reproductive value* of the population. That is the number of offspring that an individual may expect to have in the future at their current age. This vector may be scaled so that its first element is one.