THE UNIVERSITY OF SYDNEY

SCHOOL OF MATHEMATICS AND STATISTICS

AM4	Populations and Diseases	Short Assignment 2

1. *Spartina alterniflora* is a sea grass that is native to the Atlantic coast of the USA but an invasive weed on the Pacific coast. Mature *Spartina* plants form dense mats or meadows which prevent other plants growing and change the nature of the estuaries and mudflats that the plant colonises.

   Hastings *et al.* (2006) have devised a simple linear model for *Spartina* populations which they use to analyse how best to control the weed. The model is based on three different stage classes of *Spartina*: seedlings, which colonise bare mudflat, isolated plants that have not yet grown large enough to form a meadow and meadow. The model is formulated in terms of the area covered by each stage, rather than in terms of the number of plants.

   The population matrix for *Spartina* used by Hastings *et al* is:

$$\mathbf{A} = \begin{pmatrix} 0 & 0.0000646 & 0.0177 \\ 1 & 1.115 & 0 \\ 0 & 0.265 & 1.107 \end{pmatrix}$$

   (a) Draw the life cycle graph for this organism. Explain what each arc represents.

   (b) What does the difference in magnitude in the non-zero terms in the top row of $\mathbf{A}$ say about the biology of *Spartina*?

   (c) What does the 1 in the first column of the second row of $\mathbf{A}$ say about the system?

   (d) Show that the Perron-Frobenius theorem can be applied to this model.

   (e) Using MATLAB or otherwise, find the dominant eigenvalue of $\mathbf{A}$ and its corresponding left and right eigenvector. Interpret your results.

   Note: If you use the command `[WW,lambda] = eig(A)` MATLAB returns the eigenvalues as diagonal elements of the matrix `lambda` with the corresponding right eigenvectors in the matrix `WW`. The left eigenvectors can be found using the command `VV = (conj(inv(WW)))'` which produces a matrix VV whose columns contain the left eigenvectors corresponding to the right eigenvectors in the columns of `WW`.

   (f) Weed control strategies can target some or all of the three different stages. If only one stage can be controlled, which stage would be best to target? Explain your answer.

2. (a) For each of the life cycle graphs below, construct the corresponding population matrix.

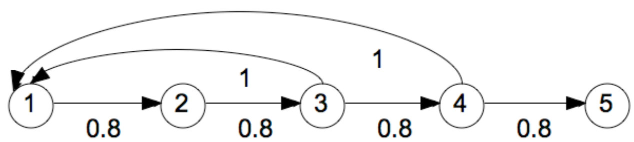
   (b) Determine whether the matrix is reducible or irreducible and if it is irreducible determine if it is primitive.

   (c) If a life cycle graph is reducible, identify and eliminate any postreproductive classes.

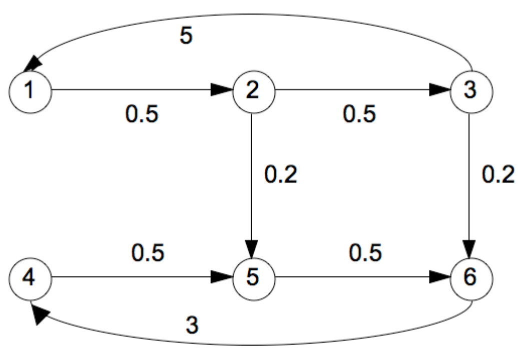
   (d) If the matrix remains reducible, divide the matrix into suitable submatrices. Identify any isolated diagonal blocks.

   (e) Find the dominant eigenvalues and corresponding eigenvectors of the matrix and its submatrices (if applicable). Interpret your results. What happens to the population as $t \to \infty$?

(i)



(ii)



(iii)