THE UNIVERSITY OF SYDNEY

SCHOOL OF MATHEMATICS AND STATISTICS

AM4	Populations and Disease	Example 2

“Birth” and death in the termite mound

A termite colony consists of one queen, one king (the mature reproductives), a large number of workers who collect food and maintain the mound and a similarly large number of nymphs which are immature reproductives that leave the mound in a large group at a particular season of the year. All eggs are laid by the queen at a rate $r(t)$. These eggs can grow into either workers or nymphs.

In 2005 David Cameron, as part of his honours project, used differential equation models to predict that the pattern of reproduction of workers versus nymphs in mound-building termites depends on whether or not the termites store food in their mounds (Cameron *et al* 2008 *Bull. Math. Biol.* **70**, 189–209).. In January 2006 we extended this model by introducing an age-dependent death rate for workers into the model. This changed the equation for the change in worker population from a linear differential equation to a McKendrick-von Foerster equation.

Let t be time and a be age of workers, both in years. The colony is established by the king and queen at $t = 0$. Initially there are no other termites in the colony. The proportion of eggs that develop into nymphs at time t is $p(t)$ so the proportion of eggs that develop into workers is $1 - p(t)$. Let $w(a, t)$ be the number of workers age a at time t. Write down the McKendrick-von Foerster equation for the worker population. Specify initial conditions and boundary conditions.

Write down the solution in integral form in terms of $r(t)$ and $p(t)$ and the age-dependent mortality $\mu(a)$.

Find an expression for the total number of workers in the colony at time t if $r(t) = R$, a constant and $p(t) = \frac{1}{2}$ when $\mu(a) = \gamma a$ where γ is a constant. What happens to the worker population as $t \to \infty$?