SCHOOL OF MATHEMATICS AND STATISTICS

AM4 | Populations and Disease | Example 5

Modelling discrete age-structured populations using Leslie matrices

Construct a table for P_i, ℓ_i and F_i for the following scenario. You may assume that any averaging that is required has already been done for you. An annually reproducing population of birds first reproduces when females are two years old. In their first reproductive season each female produces on average 6 offspring, half of which are female on average. In their subsequent breeding season each female produces 3 offspring half of which are female on average. Only 40% of the birds survive their first year; after that 80% of the survivors survive until the next season until the 5th year when all the remaining survivors die.

What is the expected proportion of the offspring which survive to the fourth year? How many female offspring on average will each female individual produce before she dies?

Construct a Leslie matrix for this scenario. This will be a 6×6 matrix. Take an initial condition of $n_i(0) = 100$ (that is every age class has 100 female individuals) and apply the Leslie matrix for a few time steps. What happens to the population? Comment on the effect of the initial condition. Graph $n_i(t)$ *vs* t for each i.

Using matlab, let the model run for about 30 seasons. You may find that the population becomes so large that calculations are unwieldy. If this is the case, divide $n_i(t)$ by the total population $N(t) = \sum_{i=1}^{a=f} n_i(t)$. You will lose information about the total size of the population by doing this, but retain information about the age distribution.

Find the eigenvalues and (right) eigenvectors of the Leslie matrix using the command

$$[\texttt{WW}, \texttt{lambda}] = \texttt{eig}(\texttt{LL});$$

where `WW` is an array whose columns are the eigenvectors and `lambda` is a diagonal array whose elements are the corresponding eigenvalues of the Leslie matrix `LL`. What is largest eigenvalue? It is real or complex? Compare the eigenvector which corresponds to the largest eigenvalue with the the scaled version of $n_i(t)$ that you have obtained by iterating the Leslie matrix. What do you notice?

Consider a change in the reproductive behaviour of the birds in the model so that each female bird produces only one female offspring in her first breeding season and, on average, half a female offspring in each breeding season thereafter. Write down a new Leslie matrix and find its largest eigenvalue and corresponding eigenvector. Interpret your results.