

Analysing travelling waves – The Spruce Budworm Model

Spruce budworm are an insect pest in the balsam fir forests of Canada. As forestry is a major industry there, budworm control is economically important. Ludwig *et al.* (1978) proposed a model for budworm population dynamics. In non-dimensionalised form it is

$$\frac{du}{dt} = ru\left(1 - \frac{u}{q}\right) - \frac{u^2}{1 + u^2} = f(u). \quad (1)$$

Show that there are up to three non-zero steady state populations and find their stability.

When the budworm model is modified to include random movement of the budworm, we get the following non-dimensionalised equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f(u). \quad (2)$$

Consider equation (2) in the region of parameter space where $f(u) = 0$ has four solutions. Let $u_1 < u_2 < u_3$ be the nonzero solutions for $f(u) = 0$.

Convert equation (2) to travelling wave coordinates and find the steady states of the resulting equation. Find the linear stability of each steady state. Sketch the $U - V$ phase plane and draw all solutions which may correspond to travelling wave solutions. (You may need to draw phase planes for several different cases.) Sketch the travelling wave profile $U(z)$ for each of these solutions and note any restrictions on parameter values.