

Errata for “Iwahori–Hecke algebras and Schur algebras of the symmetric group”

CHAPTER 1

Page 9, Corollary 1.17: \mathcal{K} should be the algebraic closure of $\mathbb{C}(\hat{q})$

CHAPTER 2

Page 17, line -2: Replace “...and one can check that $C^{*\lambda} \cong \text{Hom}_R(C^\lambda, R)$ ” with “and $\text{Hom}_R(C^{*\lambda}, R)$ is a right A -module with A -action given by $(f \cdot a)(x) = f(ax)$, for $f \in \text{Hom}_R(C^{*\lambda}, R)$, $a \in A$ and $x \in C^{*\lambda}$.”

Page 21, proof of 2.18: Delete “that $C^\nu \cong \text{Hom}_R(C^{*\nu}, R)$ as right A -modules. Therefore” and after the displayed equation add “Hence, $\dim P^\lambda \otimes_A C^{*\nu} = [\text{Hom}_R(C^{*\nu}, R) : D^\lambda] = d_{\nu\lambda}$ since $\text{Hom}_R(D^{*\mu}, R) \cong D^\mu$ whenever $D^\mu \neq 0$.”

Page 24, exercise 7(i). Delete “In addition, show that $C^\lambda \cong \text{Hom}_R(C^{*\lambda}, R)$ for all $\lambda \in \Lambda$.”

Page 25, line 3: change parenthetical remark to (the four Kazhdan–Lusztig bases $\{C_x\}$, $\{C'_x\}$, $\{D_x\}$ and $\{D'_x\}$ of $\mathcal{H}(\mathfrak{S}_n)$ are all cellular; however, this is not true for Hecke algebras of other types).

CHAPTER 3

Page 38, Warning: S^λ is the dual of the Dipper–James Specht module indexed by λ ; that is, $S^\lambda \cong (S_{DJ}^\lambda)^\circ$. One can check that $(S_{DJ}^\lambda)^\circ \cong S^{\lambda'}$ so it is necessary to replace λ with λ' when comparing our results with those of Dipper and James.

Page 41, line -1: The reduction to the case where $k = n$ is a bit of a leap. Here are more details.

Suppose $k < n$ where k is the number in the last row and the first column of t^λ . As in the second paragraph of the proof, let $\mu = (\lambda_1, \dots, \lambda_r, 1)$. Then $k = |\mu|$ and $m_\lambda = hm_\mu$ if we set $h = \sum_w T_w$, where the sum is over the elements of $\mathfrak{S}(\{k, k+1, \dots, n\})$. By the argument for the case when $k = n$ it follows that

$$m_\lambda L_k = hm_\mu L_k = h \left(\text{res}_{t^\lambda}(k) m_\mu + \sum_{\substack{\nu \vdash k, \nu \triangleright \mu \\ u, v \in \text{Std}(\nu)}} r_{uv} m_{uv} \right),$$

for some $r_{uv} \in R$. If ν is a partition of k let $\hat{\nu}$ be the partition of n obtained by appending 1^{n-k} to ν and if $u \in \text{Std}(\nu)$ let \hat{u} be the unique $\hat{\nu}$ -tableau such that $\hat{u} \downarrow k = u$. Under the natural embedding $\mathcal{H}(\mathfrak{S}_k) \hookrightarrow \mathcal{H}(\mathfrak{S}_n)$, it is easy to see that if $\nu \triangleright \mu$ and $u, v \in \text{Std}(\nu)$ then $m_{uv} = m_{\hat{u}\hat{v}}$. Notice that if $\nu \triangleright \mu$ then $\hat{\nu} > \lambda$. Therefore, returning to the last displayed equation,

$$m_\lambda (L_k - \text{res}_{t^\lambda}(k)) = h \sum_{\substack{\nu \triangleright \mu \\ u, v \in \text{Std}(\nu)}} r_{uv} m_{uv} \in \mathcal{H}^\lambda \cap \bigcap_{\nu > \lambda} \mathcal{H}^\nu \subseteq \tilde{\mathcal{H}}^\lambda.$$

Therefore, $m_\lambda L_k \equiv \text{res}_{t^\lambda}(k) m_\lambda \pmod{\tilde{\mathcal{H}}^\lambda}$ as we needed to show.

Page 44, line -6: missing bracket

Page 46, line 11: Replace l_e^λ with l_e throughout.

Page 46, Lemma 3.40: The proof of this lemma is not particularly clear. Here is a better argument:

Let $\mathbb{L}_0, \mathbb{L}_1, \dots, \mathbb{L}_k$ be the ladders which meet $[\lambda]$ and let r_a be the row index of the highest node in $\mathbb{L}_a \cap [\lambda]$, for $0 \leq a \leq k$. By definition, if $b \geq 1$ then $\lambda_b - \lambda_{b+1} < e$ because λ is e -restricted. Therefore, if (i, j) and (i', j') are two nodes in the ladder \mathbb{L}_k , for $k = l_e(i, j)$, then $(i', j') \in [\lambda]$ whenever $(i, j) \in [\lambda]$ and $i \leq i'$. That is, the ladder $\mathbb{L}_0, \dots, \mathbb{L}_k$ are ‘unbroken’ in the sense that all of the nodes which belong to the same ladder as (i, j) and appear in a later row also belong to $[\lambda]$. Hence, we see that $r_0 \geq r_1 \geq \dots \geq r_k$.

Now suppose that \mathfrak{t} is a standard tableau, not necessarily of shape λ , such that $\text{res}(\mathfrak{t}) = \text{res}(l_e^\lambda)$. To prove the Lemma we have to show that $\mathfrak{t} \triangleright l_e^\lambda$. We argue by induction on n .

When $n = 1$ then $\mathfrak{t} = l_e^\lambda = t^\lambda$ and there is nothing to prove, so suppose that $n > 1$. Let $i = \text{res}_{l_e^\lambda}(n)$ and let m be maximal such that $\text{res}_{l_e^\lambda}(m) \neq i$. Thus, recalling that $\mathbb{L}_0, \mathbb{L}_1, \dots, \mathbb{L}_k$ are the ladders which meet $[\lambda]$, n appears at the top of the ladder \mathbb{L}_k in l_e^λ and m appears at the top of \mathbb{L}_{k-1} . Let $\mu = \text{Shape}(l_e^\lambda \downarrow m)$.

As noted above, the ladders $\mathbb{L}_0, \dots, \mathbb{L}_k$ are unbroken. Therefore, μ is an e -restricted partition and $l_e^\mu = l_e^\lambda \downarrow m$. Moreover, $\mathfrak{t} \downarrow m$ is a standard tableau with residue sequence $\text{res}(l_e^\mu)$. Therefore, by induction on n ,

$$(\dagger) \quad \text{Shape}(\mathfrak{t} \downarrow r) \supseteq \text{Shape}(l_e^\mu \downarrow r) = \text{Shape}(l_e^\lambda \downarrow r),$$

for $r = 1, 2, \dots, m$, with equality throughout if and only if $\mathfrak{t} \downarrow m = l_e^\mu$. Consequently, if $\mathfrak{t} \not\supseteq l_e^\lambda$ then there must exist an integer p such that $\text{Shape}(\mathfrak{t} \downarrow p) \not\supseteq \text{Shape}(l_e^\lambda \downarrow p)$ and $m < p \leq n$. Let p be minimal with this property and set $\nu = \text{Shape}(\mathfrak{t} \downarrow p)$. For convenience, set $\lambda = (\lambda_1, \lambda_2, \dots)$, $\mu = (\mu_1, \mu_2, \dots)$ and $\nu = (\nu_1, \nu_2, \dots)$. By construction, p appears in row $r_k + n - p$ of l_e^λ . Therefore, by the minimality of p ,

$$\nu_1 + \dots + \nu_{r_k+n-p} < \lambda_1 + \dots + \lambda_{r_k+n-p} = \mu_1 + \dots + \mu_{r_k+n-p} + 1.$$

Since $\mathfrak{t} \downarrow m \supseteq l_e^\mu$ this forces $\nu_1 + \dots + \nu_{r_k+n-p} = \mu_1 + \dots + \mu_{r_k+n-p}$ and hence that $\mathfrak{t} \downarrow m = l_e^\mu$. As $m + 1, \dots, n$ occupy the lowest addable i -nodes in l_e^λ this easily implies that $\mathfrak{t} \supseteq l_e^\lambda$, giving a contradiction and completing the proof.

Page 47, line 4: $\bar{\lambda}^i$ is a *composition* of λ_i .

Page 47, line 5: g_i is the greatest common divisor of $\{[\gamma]_q^1 \in \mathcal{Z} \mid \gamma \supseteq \bar{\lambda}^i\}$. That is, we must define g_i to be an element of the ring $\mathcal{Z} = \mathbb{Z}[q, q^{-1}]$.

Page 47, Lemma 3.42: Suppose that $R = \mathcal{Z}$, that $\lambda = (\lambda_1, \dots, \lambda_k)$ is a partition of n, \dots

Page 47, line -7: On the other hand, if $\lambda \supseteq \nu$ then $(\lambda_1, \lambda_2, \dots, \lambda_k) \supseteq (\lambda_1, \lambda_1 - \mu_1, \dots, \lambda_k - \mu_k); \dots$

Page 50, exercise 18(ii): As pointed out to me by Eric Marberg, it is not true that \mathfrak{S}_μ is a subgroup of the column stabiliser of $t^\lambda d$; for example, take $\lambda = \mu = (2, 2, 1)$ and $d = (2, 3, 4, 5)$. Instead one should prove that i and j are in different rows of $t^\lambda d$ whenever they are in the same row of t^μ . This implies that $\mu' \supseteq \lambda$.

Page 53, line -7: $h_i = h_{i1}^\lambda$.

CHAPTER 4

Page 58, line 4: $\text{last}(T) = \begin{array}{|c|c|c|c|} \hline 1 & 2 & 6 & 8 \\ \hline 3 & 4 & 5 & 7 \\ \hline \end{array}$.

CHAPTER 5

Page 70, line 5: if and only if $d(\text{first}(S)) \supseteq d(\text{first}(T))$.

Page 72, line 2: then $a < c$

Page 72, before Lemma 5.12: Insert the lines “Define maps $\varphi_{T^\mu T^\omega} : M^\mu \rightarrow M^\omega$ and $\varphi_{T^\omega T^\mu} : M^\omega \rightarrow M^\mu$ by $\varphi_{T^\mu T^\omega}(m_\mu h) = m_\mu h$ and $\varphi(h) = m_\mu h$ for all $h \in \mathcal{H}$. Note that $M^\omega = \mathcal{H}$. Both $\varphi_{T^\mu T^\omega}$ and $\varphi_{T^\omega T^\mu}$ are elements of the Schur algebra.

Page 73, line 18: Note that if $x = (i, j)$ and $y = (k, l)$ then $j - i + k - l + 1$.

Page 74, line 5: Replace \mathfrak{S}_n with \mathfrak{S}_μ .

Page 75, line 1: Suppose that $\mathfrak{t} = \text{first}(T)$. Then $\pi_{\mathfrak{t}}^\mu \varphi_T = \varphi_{\mathfrak{t}} \varphi_{T^\omega T^\mu} \dots$

Page 78, line -2: B_4 should be G_4 .

Page 78, line -2: ... columns $c_1 < c_2 < \dots < c_z$

Page 81, line 15: $M_{\mathbb{F}}(k) = M(k) \otimes_R \mathbb{F}$ should be $M_{\mathbb{F}}(k) = (M(k) + \mathfrak{p}M(k))/\mathfrak{p}M(k)$.

Page 85, line 10: is well-defined

Page 86, Proof of Theorem 6.37: The last paragraph of the proof contains a gap. The proof breaks down in the case where W^λ is irreducible. The following argument corrects this.

To prove the theorem it is enough to show that $W_{\mathbb{F}}^{\mu}$ and $W_{\mathbb{F}}^{\gamma}$ are in the same block. If $\nu_p([h_{ab}^{\mu}]_{\hat{q}}) \neq \nu_p([h_{ac}^{\mu}]_{\hat{q}})$, for some nodes $(a, b), (a, c) \in [\mu]$, then we may assume that $b = 1$ and let ν be the partition obtained by removing the (a, c) -hook from μ and wrapping it back on at the bottom of the first column of $\mu \setminus R_{ac}^{\mu}$. Then $\mu \triangleright \nu$ and $W_{\mathbb{F}}^{\nu}$ appears with non-zero multiplicity in $\sum_{i>0} W_{\mathbb{F}}^{\mu}(i)$ by Theorem 5.32. Therefore, $W_{\mathbb{F}}^{\nu}$ and $W_{\mathbb{F}}^{\mu}$ are in the same block by Lemma 5.36 (and Corollary 2.22). By induction on dominance, $W_{\mathbb{F}}^{\nu}$ and $W_{\mathbb{F}}^{\gamma}$ are in the same block, so $W_{\mathbb{F}}^{\mu}$ and $W_{\mathbb{F}}^{\gamma}$ are in the same block as we wanted to show.

Finally, suppose that $\nu_p([h_{ab}^{\mu}]_{\hat{q}}) = \nu_p([h_{ac}^{\mu}]_{\hat{q}})$, whenever $(a, b), (a, c) \in [\mu]$. That is, the ν_p -valuation of the hook lengths in μ are constant along rows. Since μ is not an e -core it has at least one removable e -hook. Let R be the lowest removable e -hook in μ . That is, no other removable e -hook in μ has nodes in a lower row. Since the ν_p -valuations of the hook lengths are constant on rows, R is contained in a single column of μ . Let σ be the partition obtained by wrapping an e -hook onto the end of the first row of $\mu \setminus R$ and let ν be the partition obtained by wrapping R onto the bottom of the first column of $\mu \setminus R$. Then $\sigma \triangleright \mu \triangleright \nu$. Moreover, $W_{\mathbb{F}}^{\sigma}$ and $W_{\mathbb{F}}^{\nu}$ both appear with non-zero coefficient in $\sum_{i>0} W_{\mathbb{F}}^{\mu}(i)$ by Theorem 5.32. Therefore, the three Weyl modules $W_{\mathbb{F}}^{\sigma}$, $W_{\mathbb{F}}^{\mu}$ and $W_{\mathbb{F}}^{\nu}$ all belong to the same block by Lemma 5.36. By induction $W_{\mathbb{F}}^{\nu}$ and $W_{\mathbb{F}}^{\gamma}$ are in the same block since $\mu \triangleright \nu$. Hence, $W_{\mathbb{F}}^{\mu}$ and $W_{\mathbb{F}}^{\gamma}$ are in the same block, completing the proof.

CHAPTER 6

Page 98, Prop. 6.1: Replace “ $\nu_i \triangleright \nu_j$ ” with $\nu_j \triangleright \nu_i$. In the proof replace “earlier row” with “later row”.

Page 105, line -5: “good” should be “removable”.

Page 119, line 20: delete not.

Page 128, paragraph -1: The paper of Martin and Russel contained a serious gap and even missed some cases. Fortunately, Matthew Fayers has now proved that the decomposition numbers of the blocks of weight 3 are at most 1 when $p > 3$; see “Decomposition numbers for weight three blocks of symmetric groups and Iwahori–Hecke algebras”, *Trans. Amer. Math. Soc.*, **360** (2008), 1341–1376. In a sequel to this paper, Fayer’s proved James’ conjecture for blocks of the Iwahori–Hecke algebra of weight 4; see his paper “James’s conjecture holds for weight four blocks of Iwahori–Hecke algebras”, *J. Algebra*, **317** (2007), 593–633.

Page 135, Note added in proof: Grojnowski and Vazirani [C,E] prove Conjecture 6.54 only up to a permutation of the multipartitions which index the simple modules of the Ariki–Koike algebras. Ariki has now proved that this permutation is trivial, thus establishing Conjecture 6.54; see his paper “Proof of the modular branching rule for cyclotomic Hecke algebras”, *J. Algebra*, **306** (2006), 290–300.

Grojnowski’s [B] proof of the classification of the blocks of the Ariki–Koike algebras (conjecture 6.53) was incomplete. The conjecture has now been proved by Lyle and Mathas; see “Blocks of cyclotomic Hecke algebras”, *Adv. Math.*, **216** (2007), 854–878.

Page 135: line 11: A should be B.

Page 135: line -11: E should be D.

APPENDIX A

Page 142, line -11: $\pi^G : M \longrightarrow N$.

Page 142, line 2: The ideals $\text{Rad } P_i \oplus \bigoplus_{j \neq i} P_j$ are only some of the maximal ideals of A . The argument should be replaced with the following.

Write $A = P_1 \oplus \cdots \oplus P_k$ as a direct sum of principal indecomposable modules. Then the modules $\text{Rad } P_i \oplus \bigoplus_{j \neq i} P_j$, for $i = 1, 2, \dots, k$, are maximal ideals of A . The intersection of these ideals is $\text{Rad } P_1 \oplus \cdots \oplus \text{Rad } P_k = \text{Rad } A$; hence, the Jacobson radical of A is contained in $\text{Rad } A$.

Conversely, suppose that M is a maximal ideal of A . Then A/M is simple so $(\text{Rad } A)(A/M)$ is either 0 or A/M . However, $\text{Rad } A$ is nilpotent so $(\text{Rad } A)^n = 0$ for some n ; therefore, $(\text{Rad } A)(A/M) = 0$ since otherwise $A/M = 0$. In particular, this shows that $\text{Rad } A \subseteq (\text{Rad } A)A \subseteq M$. Therefore, $\text{Rad } A$ is contained in every maximal ideal of A ; consequently, $\text{Rad } A$ is contained in the Jacobson radical of A . This completes the proof.