

**APPLIED MATHEMATICS IV:
MODERN ASYMPTOTICS AND PERTURBATION THEORY
2010**

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Differential equations model most natural phenomena we know. Yet their solutions can be notoriously difficult to understand. In place of “exact” solutions, a rich array of *asymptotic* methods have been developed to qualitatively understand the solutions. These are based on either the intrinsic variables or an external parameter in the problem being large or small. The field is vast and ranges from the fundamental theory of asymptotic expansions and perturbation methods, developed by Poincaré for the study of the solar system, to modern advanced techniques which deal with cases where conventional asymptotics fails. This course will include asymptotics of boundary layers, WKB and multiscale methods (depending on the background of the class) and modern techniques developed to model dendritic growth (such as snowflakes), fluid flow (existence of solitary waves), and the onset of chaos. The only background needed is the basic theory of differential equations and complex analysis.

ASSESSMENT

Assignments: 40% ; Exam: 60%

There will be two assignments, to be given out approximately in weeks 4 and 8 respectively. The exam is a take-home exam, to be done over a three-day period.

REFERENCES

Some recommended books:

- (1) C. Bender and S. Orszag, *Advanced mathematical methods for scientists and engineers*, International Series in Pure and Applied Mathematics. McGraw-Hill Book Co., New York, 1978. Reprinted by Springer-Verlag, New York, 1999. **Placed in Mathematics Library Reserve.**
- (2) A. Nayfeh, *Perturbation methods*. Reprint of the 1973 original. Wiley Classics Library. Wiley-Interscience [John Wiley & Sons], New York, 2000.
- (3) F. W. J. Olver, *Asymptotics and Special Functions*. Reprint of the 1974 original [Academic Press, New York]. AKP Classics. A K Peters, Ltd., Wellesley, MA, 1997.

OUTLINE OF TOPICS

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| 1. Introduction | 4. Multiple-scales expansions |
| 2. Irregular singular points of ODEs | 5. WKB theory |
| 3. Boundary layer theory | 6. Asymptotics beyond all orders |

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