

Optimal Approximation

We use MAPLE to 20 digits on $F(x) := xe^{-x}Ei(x)$.

$$x = 10$$

$$\begin{aligned} F(10) &= 1.1314702047341077803 \\ \sum_{n=0}^9 \frac{n!}{10^n} &= 1.13159008000000000000 \\ \sum_{n=0}^{10} \frac{n!}{10^n} &= 1.13195296000000000000 \end{aligned}$$

$$x = 100$$

$$\begin{aligned} F(100) &= 1.0102062527748357112 \\ \sum_{n=0}^{99} \frac{n!}{100^n} &= 1.0102062527748357112 \\ \sum_{n=0}^{260} \frac{n!}{100^n} &= 1.0108296120102260952 \\ \sum_{n=0}^{281} \frac{n!}{100^n} &= 732496.06921461904157 \end{aligned}$$