Exercise 1 Solutions

1. (a) The mean is 19.917 and the standard deviation is 2.968. The estimated standard error of the mean is $s/\sqrt{n} = 2.968/\sqrt{12} = 0.8569$.

(b) The only assumptions needed to use the above formula for the standard error of the mean is that the observations are independent and identically distributed.

(c) The 5 number summary is

$(16, 17.5, 19.5, 22, 26)$.

The boxplot is reasonably symmetric and so the data appear to be consistent with the normal assumption.

(d) A 95% C.I. for the population mean is based on the $t$ statistic as the population variance is unknown. The C.I. is

$19.917 \pm 2.201 \times 0.8569$

i.e. $19.917 \pm 1.886$.

(e) Test $H_0 : \mu = 17$ against $H_1 : \mu > 17$.

$$t = \frac{\bar{x} - 17}{s/\sqrt{n}} = 3.404$$

$p$-value = $P(t_{11} \geq 3.404) = 0.00294$. Thus there is strong statistical evidence to support the claim that the mean is greater than 17.

Assumptions: The observations are independent and have been drawn from a population that can be modelled by a normal random variable.

2. (a) The correlation is $1/\sqrt{9 \times 2} = 0.2357$.

(b) The variance is 204.

(c) The standard deviation is $\sqrt{16} = 4$. 