Exercise 4 Solutions

1. (a) To test $H_0: \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0$ use

$$f = \frac{(S_1 - S_0)/4}{S_0/26} = 1.553.$$  

$p$-value $= P(F_{4,26} \geq 1.553) > 0.10.$

Thus the data are consistent with the null hypothesis.

(b) To test $H_0 : \beta_4 = 0$ use the $p$-value from the lm summary for lm2. $p = 0.2695$ so we can conclude that $x_4$ can be dropped.

(c) A point is a high leverage point if $h_{ii} > 2\times 4/35 = 0.228$. There are 3 high leverage points Western Cas and Surety ([10]), Nevada Power ([18]) and So. Union ([35]). None of these points are outliers.

(d) A 95% CI for the expected stock price is

$$23.087 \pm 2.0395 \times 1.6668$$

i.e. $23.087 \pm 3.399.$

2. (a) An estimate for $\sigma^2$ is $\hat{\sigma}^2 = 0.674/(8 - 3) = 0.1348.$

(b) A 95% CI for $\beta_1$ is

$$-0.09 \pm t_{5}(0.975)\hat{\sigma}\sqrt{0.00147}$$

i.e. $-0.09 \pm 0.036.$

(c) Test $H_0 : \beta_2 = 0$ against $H_1 : \beta_2 > 0$. The test statistic is

$$\tau = \frac{0.083}{\sqrt{0.1348 \times 0.00359}} = 3.773.$$

$p = P(t_5 \geq 3.773) = 0.007.$ Thus we have strong evidence to support the claim that $\beta_2$ is positive.

(d) From (b) and (c) we can see that neither $x_1$ nor $x_2$ can be dropped from the model. A 95% CI for $\beta_0$ is $13.109 \pm 2.200$ so we see that the model cannot be simplified.

(e) An estimate for the average fuel consumption when $x_1 = 40$ and $x_2 = 20$ is obtained by substituting into the regression equation. The estimate is 11.169 tons.