1. Model \( Y_{ij} = \mu + \alpha_i + \epsilon_{ij} \), \( \epsilon_{ij} \sim N(0, \sigma^2) \), where \( Y_{ij} \) denotes the \( j \)th observation on brand \( i \).

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>SS</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brands</td>
<td>3</td>
<td>29860.4</td>
<td>9953.467</td>
<td>16.42</td>
</tr>
<tr>
<td>Residual</td>
<td>16</td>
<td>9698.4</td>
<td>606.15</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
<td>39558.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Test \( H_0: \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 \). The \( p \)-value for the test is

\[ P(F_{3,16} \geq 16.42) < 0.001, \]

so there is strong evidence that the mean number of drives is not the same across the 4 brands.

(b) 95% Scheffé simultaneous C.I.s use \( \sqrt{3F_{3,16}(0.95)} = 3.118 \). The C.I.s are

A vs C

\[ (1268/5 - 1209/5) \pm 3.118 \times \sqrt{606.15(1/5 + 1/5)} = 11.8 \pm 48.55. \]

B vs average of A and C

\[ (1532/5 - (1268+1209)/10) \pm 3.118 \times \sqrt{606.15(1/5 + (-0.5)^2/5 + (-0.5)^2/5)} = 58.7 \pm 42.04. \]

(c) The three orthogonal contrasts are \( \alpha_1 - \alpha_3 \), \( \alpha_2 - \alpha_4 \), and \( \frac{1}{2}(\alpha_1 + \alpha_3) - \frac{1}{2}(\alpha_2 + \alpha_4) \).

The component SS are:

A vs C

\[ \frac{(1268/5 - 1209/5)^2}{1/5 + 1/5} = 348.1 \]

B vs D

\[ \frac{(1683/5 - 1532/5)^2}{1/5 + 1/5} = 2280.1 \]

(A,C) vs (B,D)

\[ \frac{((1683 + 1532)/10 - (1268 + 1209)/10)^2}{0.5^2/5 + (-0.5)^2/5 + 0.5^2/5(0.5)^2/5} = 27232.2. \]
2. (a) The treatment SS is

\[ 320.33^2/5 + 259.21^2/4 + 175.43^2/3 + 152.84^2/3 - 907.81^2/15 = 423.7016. \]

<table>
<thead>
<tr>
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<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>3</td>
<td>423.7016</td>
<td>141.2339</td>
<td>11.202</td>
</tr>
<tr>
<td>Residual</td>
<td>11</td>
<td>138.6867</td>
<td>12.6079</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>14</td>
<td>562.3883</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thus the p-value for testing the claim that the average percentage is the same across the 4 groups is \( P(F_{3,11} \geq 11.202) = 0.0011 \). Conclude that the percentage of beetles surviving does depend on the density.

(b) The linear contrast coefficients are \( n_i(x_i - \bar{x}) \) where \( \bar{x} = 37 \) in this case. The coefficients are \( c_i : -160, -68, 39, 189 \).

The linear component of the Treatment SS is

\[
\frac{(-160 \times 320.33/5 - 68 \times 259.21/4 + 39 \times 175.43/3 + 189 \times 152.84/3)^2}{(-160)^2/5 + (-68)^2/4 + 39^2/3 + 189^2/3} = 403.928.
\]

Test \( H_0 : E(Y_{ij}) \) varies linearly with \( x_i \):

\[
f = \frac{(423.7016 - 403.928)/2}{138.6867/11} = 0.784.
\]

\[
p = P(F_{2,11} \geq 0.784) = 0.481,
\]

thus the data are consistent with a simple linear regression relationship.