1. Let $Z \sim N(0, 1)$ and $Z_1 \sim N(1, 1)$.

   (a) Find the following probabilities:
   \[
P(Z \leq 1.25); \quad P(0.5 < Z_1 \leq 1.23);
   P(t_3 \leq 2.353); \quad P(|t_5| \leq 1.476).
   \]

   (b) In each case find $c$ such that
   \[
P(t_8 > c) = 0.05; \quad P(|t_7| \geq c) = 0.50;
   P(Z < c) = 0.05; \quad P(|Z| \geq c) = 0.05.
   \]

2. A chemical process has produced, on the average, 800 tons of chemical per day. The daily yields for the past week are 785, 805, 790, 793, and 802 tons. We wish to determine whether the data indicate that the average yield is less than 800 tons and hence that something is wrong with the process.

   (i) State the null and alternative hypothesis.

   (ii) What assumptions must be satisfied in order to carry out the test?

   (iii) Find the test statistic and calculate the $p$-value.

   (iv) What is your conclusion?

3. A USA supreme court case in 1965 concerned a panel of 100 potential jurors, of whom 8 were black. The panel was selected from a population of whom about 26% were black. Do you think the selection is random? (State formally the hypothesis being tested here and the evidence against it in terms of a $p$-value)
Computer Exercises week 1

Note: \texttt{pnorm}, \texttt{pt} give distribution functions; \texttt{qnorm}, \texttt{qt} give quantiles.

1. Let $Z \sim \mathcal{N}(0, 1)$ and $Z_1 \sim \mathcal{N}(1, 1)$. Use R to
   
   (a) find the following probabilities:
   
   \[
   P(Z \leq 1.25); \quad P(0.5 < Z_1 \leq 1.23);
   \]
   
   \[
   P(t_3 \leq 2.353); \quad P(|t_5| \leq 1.476).
   \]

   (b) find $c$ such that

   \[
   P(t_8 > c) = 0.05; \quad P(|t_7| \geq c) = 0.50;
   \]

   \[
   P(Z < c) = 0.05; \quad P(|Z| \geq c) = 0.05.
   \]

2. The dataset \texttt{amp} are the results of measurements made using two methods on 15 pairs of tablets to determine the dosage of aspicillin. Analyze the data to determine if there is a systematic difference between two methods.

   (a) Type \texttt{amp} to have a look the dataset.

   (b) State the null and alternative hypotheses.

   (c) Create two vectors \texttt{method.A}, \texttt{method.B} whose entries correspond to two columns of the data.

   (d) Calculate the mean difference $\bar{d}$ and its standard deviation $s_d$.

   (e) Calculate the observed value of the paired $t$-test statistic (use \texttt{length(method.A)} to get the sample size $n$) and then the $p$-value.

   (f) Use the built in command \texttt{t.test} to confirm the $p$-value.

   (g) State your conclusions, and explain in words what assumptions must be satisfied in order for the procedure you used to analyze the data to be valid.