Random matrices and the six-vertex model
Pavel Bleher
Indiana University–Purdue University Indianapolis
Joint work with Vladimir Fokin, Karl Liechty, and Thomas Bothner
We will review various results on the large $N$ asymptotic behavior of the partition function of the six-vertex model with domain wall boundary conditions and its relation to random matrices and orthogonal polynomials.

Random remarks on computing gap probabilities: methods, software, applications, open problems
Folkmar Bornemann
TU Munich
We review five years of computing gap probabilities in RMT based on evaluating operator determinants. We look at some recent applications, discuss issues of assessing accuracies and mention open problems.

Biorthogonal ensembles with two-particle interactions
Tom Claeys
UC Louvain
I will talk about determinantal point processes on $[0, \infty)$ of the form
\[
\frac{1}{Z_n} \prod_{1 \leq i < j \leq n} (\lambda_i - \lambda_j) \prod_{1 \leq i < j \leq n} (\lambda_i^\theta - \lambda_j^\theta) \prod_{j=1}^n w(\lambda_j) d\lambda_j,
\]
with $\theta > 1$. I will show that the biorthogonal polynomials associated to such models satisfy a recurrence relation and a Christoffel-Darboux formula if $\theta$ is rational, and that they can be characterized in terms of non-standard $1 \times 2$ Riemann-Hilbert problems. If $w(\lambda) = e^{-nV(\lambda)}$, I will also construct the equilibrium measure associated to the model in the one-cut case with and without hard edge. The talk will be based on joint work with Stefano Romano.

Semi-classical orthogonal polynomials and the Painlevé equations
Peter Clarkson
University of Kent
In this talk I shall discuss the relationship between the Painlevé equations and orthogonal polynomials with respect to semi-classical weights. It is well-known that orthogonal polynomials satisfy a three-term recurrence relation. I will show that for some semi-classical weights, the coefficients in the recurrence relation can be expressed in terms of Wronskians that arise in the description of special function solutions of a Painlevé equation. The orthogonal polynomials discussed will include
Asymptotic analysis of the partition function in the cubic random matrix model

Alfredo Deaño
KU Leuven / Universidad Carlos III de Madrid
Joint work with Pavel M. Bleher

We present results on the large $N$ expansion of the free energy and the partition function of a unitary random matrix model with weight $w(z) = e^{-NV(z)}$, where $V(z) = z^2/2 - uz^3$ is a perturbation of GUE, and $u > 0$ is a real parameter. For small enough $u$, the free energy $F_N(u)$ can be expanded in powers of $N^{-2}$, in the so called topological expansion which goes back to the work of Brzin, Itzykson, Parisi and Zuber. Close to an explicit critical value $u = u_c$, it is necessary to consider a double scaling limit, and the asymptotic behavior of the free energy has a non-trivial correction with respect to the regular case, given in terms of a certain family of solutions of the Painlevé I differential equation.

Central limit theorems for biorthogonal ensembles with a recurrence

Maurice Duits
Stockholm University

In this talk I will report on a joint work with Jonathan Breuer (HUJI) in which we study fluctuations of linear statistics corresponding to smooth functions for certain biorthogonal ensembles. We study those biorthogonal ensembles for which the underlying biorthogonal family satisfies a finite term recurrence and describe the asymptotic fluctuations using right limits of the recurrence matrix. As a consequence, we show that whenever the right limit is a Laurent matrix, a Central Limit Theorem holds. I will also discuss the implications for orthogonal polynomial ensembles. In particular, we obtain a Central limit theorem for the orthogonal polynomial ensemble associated with any measure belonging to the Nevai Class of an interval. Our results also extend previous results on Unitary Ensembles in the one-cut case. Finally, I will further illustrate our results by discussing Central Limit Theorems for the Hahn ensemble for lozenge tilings of a hexagon and for the Hermitian two matrix model.

Finding the (numerical) rank: a randomized approach

Ioanna Dumitriu
University of Washington
Joint work with Jim Demmel, Olga Holtz, Grey Ballard, and Chris Melgaard

When it comes to constructing rank-revealing factorizations, a bit of randomization can go quite far. The signal-processing community has made great use of FFT-matrix sampling, as well as other techniques, to find the largest singular values of a very large matrix. Such algorithms are much faster than the $O(n^3)$ deterministic ones; however, this relies on the assumption that the rank is small compared to the size of the matrix. In 2007, we proposed a new randomized rank-revealing factorization that works when the rank is a fraction of the size of the matrix; since then, we have improved the result, as well as used it to obtain a communication-optimal nonsymmetric eigenvalue decomposition algorithm. The results are based on (relatively basic) random matrix theory.
On an integral geometry inspired method for conditional sampling from Gaussian ensembles

Alan Edelman
MIT
Joint work with Oren Mangoubi

As an example problem, consider a numerical simulation that calculates the distribution of the second eigenvalue from a classical random matrix ensemble conditioned on knowledge of the first eigenvalue. A naive Monte Carlo approach requires throwing away many samples often creating serious inefficiencies. In this talk we show how integral geometry inspires techniques for conditional probabilities. These techniques can be used any time there is a conditional statistic based on normal random variables. We will also demonstrate the power of Julia in Random Matrix Theory and may say some words about recent investigations into the early numerical experiments in Random Matrix Theory.

Asymptotics of spacing distributions

Peter Forrester
The University of Melbourne

The topic of spacing distributions is one of the oldest in random matrix theory as it applies to theoretical physics. Nonetheless, there has been progress on a number of fronts in the last few years. I’ll survey new results which relate to asymptotic behaviours.

Fluctuations of the extremal eigenvalues of Gaussian complex covariance matrices

Adrien Hardy
KTH
Joint work with Steven Delvaux, Walid Hachem and Jamal Najim

We study the extremal eigenvalues of large Gaussian complex covariance matrices, and more precisely their fluctuations around the edges of the bulk’s connected components. For generic cases, we obtain Tracy–Widom fluctuations or relatives, but we will also discuss about more exotic phenomena.

Universality in orthogonal polynomials with respect to oscillatory weights

Daan Huybrechs
KU Leuven

We discuss polynomials orthogonal with respect to an oscillatory weight. Particular families of such polynomials exhibit interesting asymptotic behaviour as the frequency of the oscillations increases: the polynomials tend to a product of lower-degree orthogonal polynomials. These lower-degree polynomials are orthogonal with respect to a simpler, non-oscillatory weight, with examples including the classical Laguerre and Hermite polynomials. This large-frequency behaviour is distinctly different from the large-degree asymptotics of the original orthogonal polynomials. We determine and classify the possible simplified weights that may appear in the case of complex exponential weight functions, and we relate our results to the linear and non-linear steepest descent analysis of oscillatory integrals and Riemann–Hilbert problems.
Singular values of products of random matrices

Arno Kuijlaars
KU Leuven
Joint work with Lun Zhang [2]

Recently, Akemann et al. [1] showed that squared singular values of products of random matrices with independent complex Gaussian entries give rise to a determinantal point process whose correlation kernel is given in terms of Meijer G-functions. The determinantal point process is in fact a multiple orthogonal polynomial ensemble. We describe some of the properties of this new class of multiple orthogonal polynomials. For the case of a product of two matrices they lead to multiple orthogonal polynomials with modified Bessel weights that were first studied by Van Assche and Yakubovich in [3].


2D Coulomb gas by orthogonal polynomials

Seung–Yeop Lee
University of South Florida

The 2D Coulomb gas that comes from normal random matrices is characterized by the kernel function. I will present some asymptotic calculation of the kernel for some simple normal random matrix models.

The semiclassical sine-Gordon equation and rational solutions of Painlevé-II

Peter Miller
University of Michigan
Joint work with Robert Buckingham (Cincinnati)

We formulate and study a class of initial-value problem for the sine-Gordon equation in the semiclassical limit. The initial data parametrizes a curve in the phase portrait of the simple pendulum, and near points where the curve crosses the separatrix a double-scaling limit reveals a universal wave pattern constructed of superluminal kinks located in the space-time along the real graphs of all of the rational solutions of the inhomogeneous Painlevé-II equation. The kinks collide at the real poles, and there the solution is locally described in terms of certain double-kink exact solutions of sine-Gordon.

This study naturally leads to the question of the large-degree asymptotics of the rational solutions of Painlevé-II themselves. In the time remaining we will describe recent results in this direction, including a formula for the boundary of the pole-free region, strong asymptotics valid also near poles, a weak limit formula, and planar and linear densities of complex and real poles.
Spectral shock waves in dynamical random matrix theories

Maciej Nowak
Jagellonian University

We obtain several classes of non-linear partial differential equations for various random matrix ensembles undergoing Brownian type of random walk. These equations for spectral flow of eigenvalues as a function of dynamical parameter ("time") are exact for any finite size $N$ of the random matrix ensemble and resemble viscous Burgers-like equations known in the theory of turbulence. In the limit of infinite size of the matrix, these equations reduce to complex inviscid Burgers equations, proposed originally by Voiculescu in the context of free processes. We identify spectral shock waves for these equations in the limit of the infinite size of the matrix, and then we solve exact, finite $N$ nonlinear equations in the vicinity of the shocks, obtaining in this way universal, microscopic scalings equivalent to Airy, Bessel and cuspoid kernels. Finally, we show that similar but hidden Burgers-like structures appear (surprisingly) also in nonhermitian random matrix models, e.g. in the Ginibre-Girko ensemble.

Random matrix theory and universal perfect transmission in opaque media

Raj Rao Nadakuditi
University of Michigan

Materials such as eggshells, turbid water and white paper are considered opaque because scattering frustrates the passage of light through such media. Consequently, only a small portion of the incident light will emerge through the medium. Surprisingly, it turns out that it "nearly always" possible to engineer a wavefront, tailored specifically to the medium, that will achieve near perfect transmission. In other words, the material will behave as though it is transparent to this specific wavefront.

We use random matrix theory to study this phenomena and show that the theoretical predictions agree remarkably well with high-precision numerical simulations for systems containing hundreds of thousands of scatterers. Finally, we show why we might expect perfect transmission in nearly all sparse yet opaque random media.

Near-extreme eigenvalues and the first gap of Hermitian random matrices

Grégoire Schehr
Université de Paris–Sud
Joint work with Anthony Perret

We study the phenomenon of “crowding” near the largest eigenvalue $l_{\text{max}}$ of random $N \times N$ matrices belonging to the Gaussian Unitary Ensemble (GUE) of random matrix theory. We focus on two distinct quantities: (i) the density of states (DOS) near $l_{\text{max}}$, $\rho_{\text{DOS}}(r,N)$, which is the average density of eigenvalues located at a distance $r$ from $l_{\text{max}}$, and (ii) the probability density function (PDF) of the gap between the first two largest eigenvalues, $p_{\text{GAP}}(r,N)$. In the edge scaling limit where $r = O(N^{-1/6})$, which is described by a double scaling limit of a system of unconventional orthogonal polynomials, we show that $\rho_{\text{DOS}}(r,N)$ and $p_{\text{GAP}}(r,N)$ are characterized by scaling functions which can be expressed in terms of the solution of a Lax pair associated to the Painlevé XXXIV equation. This provides an alternative and simpler expression for the gap distribution, which was recently studied by Witte, Bornemann and Forrester in *Nonlinearity* 26, 1799 (2013). Our expressions allow to obtain precise asymptotic behaviors of these scaling functions both for small and large arguments.
Continuous spectra for sparse random graphs
Arnab Sen
University of Minnesota
Joint work with Charles Bordenave and Balint Virag

The limiting spectral distributions of many sparse random graph models are known to contain atoms. But do they also have some continuous part? In this talk, I will give affirmative answer to this question for several widely studied models of random graphs including Erdos-Renyi random graph $G(n, c/n)$ with $c > 1$, random graphs with certain degree distributions and supercritical bond percolation on $Z^2$. I will also present several open problems.

S-property in polynomial external fields
Guilherme L. F. Silva
KU Leuven
Joint work with Arno Kuijlaars

Curves in the complex plane that satisfy the S-property were first introduced by Stahl and they were further studied by Gonchar and Rakhmanov in the 1980s. Rakhmanov recently showed the existence of curves with the S-property in a harmonic external field by means of a max-min variational problem in logarithmic potential theory [4].

Rakhmanov’s work is done in a fairly general setting, which however does not include the important special case of an external field $\text{Re} V$ where $V$ is a polynomial of degree $\geq 2$. This case is motivated by its connection with polynomials orthogonal with respect to the varying weights $e^{-nV}$, see [1].

In this talk, based on [2], we plan to give a general overview on the proof of the existence of a curve with the S-property in the external field $\text{Re} V$ within the collection of all curves that connect two or more pre-assigned directions at infinity in which $\text{Re} V \to +\infty$.


Monte Carlo methods and universality in numerical algorithms
Tom Trogdon
New York University

Pfrang et al. used systematic Monte Carlo simulations of random matrices when they asked the question: “How long does it take to compute the eigenvalues of a random symmetric matrix?” Their results demonstrate universality in the fluctuations of the computation time for various random matrix ensembles. This work led to two natural questions. The first is a question of how to sample unitary invariant ensembles (UIEs) when the entries are no longer independent (i.e. not GUE). The second is a question of the pervasiveness of universality in numerical algorithms. In this talk, I will describe a method for the sampling UIEs that makes explicit use of the Riemann–Hilbert problem
for orthogonal polynomials. I will also demonstrate universality in the fluctuations for a wider class of algorithms including the conjugate gradient, GMRES and genetic algorithms.

Nonintersecting Brownian motions on the circle and discrete Gaussian orthogonal polynomials

Dong Wang
National University of Singapore
Joint work with Karl Liechty

In this talk I will discuss the relation between the nonintersecting Brownian motions on the circle, where \( n \) particles start from a common point and end at a common point, after time \( T \), and the discrete Gaussian orthogonal polynomials, i.e., discrete orthogonal polynomials with respect to the weight \( \exp(-x^2) \). The nonintersecting Brownian motions on the circle share universal asymptotic properties like the limiting Sine, Airy and Pearcey processes with the nonintersecting Brownian motions on the real line, and have new interesting quantities like the winding number of particles. As the particle number \( n \) goes to infinity, the asymptotic behavior of the model shows different features for small \( T \) and big \( T \). I will show the relation between the asymptotic behavior of the nonintersecting Brownian motion model and the asymptotic properties of the discrete Gaussian orthogonal polynomials.

Eigenvalue densities for the Gaussian \( \beta \) Ensembles

Nicholas Witte
The University of Melbourne
Joint work with Peter Forrester and Anas Rahman

There are a number of tools available to compute the density of eigenvalues for general \( \beta \) in the Gaussian ensembles, each with their own advantages and drawbacks. In the loop equation formalism one can compute the \( 1/N \) expansion of the resolvent for the Gaussian \( \beta \) ensemble up to a fixed truncation order for general \( \beta \), as it is known rigorously that such an analytic expansion is valid. The resolvent expansion allows the moments of the eigenvalue density to be computed exactly up to a certain moment power and the smoothed density to be expanded up to an equivalent truncation order. In contrast at the special couplings of even, positive and real \( \beta \), and by implication of duality at the well-studied cases of \( \beta = 1, 2 \) and 4, there are exact characterisations of both the resolvent and the moments. This knowledge allows for the corresponding expansions described above to be extended, in some recursive form at least, to arbitrary order. For example we give fifth order linear differential equations for the density and resolvent at \( \beta = 1 \) and 4, which complements the known third order linear differential equations for these quantities at \( \beta = 2 \). From these equations one can establish an exact and recursive form for the large \( N \) expansion of the resolvent.

Universality and critical behaviour in the chiral two-matrix model

Lun Zhang
Fudan University
Joint work with Steven Delvaux and Dries Geudens

In this talk, we are concerned with the chiral two-matrix model with polynomial potential functions \( V \) and \( W \), which was introduced by Akemann, Damgaard, Osborn and Splittorff. We show that the squared singular values of each of the individual matrices in this model form a determinantal point process with correlation kernel determined by a matrix-valued Riemann–Hilbert problem. The size of the Riemann–Hilbert matrix depends on the degree of the potential function \( W \) (or \( V \) respectively). For the case where \( W(y) = y^2/2 + \alpha y \) is quadratic, we derive the large \( n \)-asymptotics.
of the Riemann–Hilbert problem by means of the Deift-Zhou steepest descent method. This proves
universality in this case. Finally we show that if also \( V(x) = x \) is linear, then a multi-critical limit
of the kernel exists which is described by a \( 4 \times 4 \) matrix-valued Riemann–Hilbert problem associated
to the Painlevé II equation \( q''(x) = xq(x) + 2q^3(x) - \nu - 1/2 \), which can be viewed as the chiral
analogue of a recent result by Duits and Geudens.