\[ c_k = \frac{1}{2\pi i} \oint \frac{1}{z^k} \, dz = -\frac{d}{dz} \left( \frac{1}{z} \right) \]

\[ = -\frac{1}{2\pi i} \oint f(t) \frac{1}{t^k} \, dt \]

\[ = \frac{1}{2\pi i} \oint f(t) \frac{1}{t^{k-1}} \, dt \]

\[ = \frac{1}{2\pi i} \oint \]
Suppose \( \tilde{g} \) is analytic in \( B \) & \( \tilde{g} = f \) in \( A \cap B \), then

\[
g - \tilde{g} = 0 - 0 = 0 \quad \text{in} \quad A \cap B,
\]

therefore

\[
g - \tilde{g} = \sum_{k=0}^{\infty} \frac{0}{k!} z^k
\]

\[\Rightarrow\quad g = \tilde{g} \quad \text{in} \quad B\]
consider $f - g$ on $\Gamma$, there all derivatives of $f - g$ along $\Gamma$ are 0:

$$\frac{(f(z+h) - g(z+h)) - (f(z) - g(z))}{h} = 0$$

Therefore all derivatives are zero along curve $\Gamma$.

Therefore

$$\frac{(f(z) - g(z))}{h} \text{ at } z = z_0 \Rightarrow \lim_{h \to 0} \frac{(f(z_0+h) - g(z_0+h)) - (f(z_0) - g(z_0))}{h} = 0$$

$\Rightarrow f - g$ is zero in circle around $z_0$.

$\Rightarrow f - g$ is zero in $A$. 
Therefore $\phi$ is unique analytic ant
\[ \phi_+ + \phi_- = \phi \]
both analytic in annulus

\[ \phi_+(z) + \phi_-(z) = \phi(z) \]
on unit circle

\[ \Rightarrow \text{equal in all of annulus} \]