AMH5 Assignment 2

Due Thu 26 Oct 2017 (week 12) by 5pm.

Please email me a pdf of your answers and your MATLAB code and instructions on how to run it. (Alternatively, you can hand me a paper document, but please still email the MATLAB code and how to run it.)

Problem 1: Numerically simulating Turing patterns with a PDE

Consider the PDE system

$$\begin{aligned} u_t &= \gamma f(u, v) + u_{xx}, \\ v_t &= \gamma g(u, v) + d v_{xx} \end{aligned}$$

with no-flux boundary conditions on the domain $x \in [0, 1]$, where

$$\begin{aligned} f(u, v) &= a - u + u^2 v, \\ g(u, v) &= b - u^2 v \end{aligned}$$

for some constants γ, a, b, and d.

In class, we derived conditions for diffusion-driven instabilities for this PDE system. Using these conditions, find an example of parameters a, b, and γ, such that Turing patterns, i.e., diffusion-driven instabilities arise for some diffusion ratios d.

Choose some initial conditions that are close to, but not right on, the constant steady state (u_0, v_0).

(a) Use the Matlab PDE solver 'pdepe' to simulate this system (using your parameters above) for the case when the diffusion ratio $d = 1$. What is the long-term behaviour of the system? Is the system stable or unstable?

(b) Using the same parameters above (except for d), choose some $d > 1$ for which Turing patterns arise and simulate the system using the 'pdepe' function in MATLAB. What is the long-term behaviour of the system?

Problem 2: Agent-based model of the system

Formulate an agent-based model (in 1-D) of the PDE system in Problem 1. You will need to include two populations of agents, random walks (for diffusion), and probabilistic rates of generation and death of individual agents.

Can you get Turing patterns using the agent-based model? Compare the long-term behaviour of the agent-based model with the long-term behaviour of the PDE system simualted in Problem 1b.

Be innovative and ask me questions in class or out of class.