Discussion question

Consider the following 1-D reaction-diffusion equation for $u(x,t)$:

$$u_t = u_{xx} + u(u-\delta)(1-u),$$

where $0 < \delta < 1$.

1. Transform the equation into travelling wave coordinates, $U(z) = u(x,t)$ for $z = x - ct$, where $c > 0$. Linearise about each of the equilibria in the $U - U_z$ phase plane and calculate the eigenvalues.

2. For a certain wavespeeds c, there exists a travelling wave solution with $U \to 1$ as $z \to -\infty$ and $U \to 0$ as $z \to \infty$. (You don't need to show this.) Draw the heteroclinic trajectory between the relevant equilibria that defines the traveling wave solution. Be sure to make sure that the trajectory agrees with the nullclines in the phase plane.