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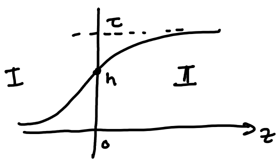
$$u(x,t) = f(x-ct)$$
$$\frac{\partial f}{\partial t} = -cf$$
$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 u}{\partial x^2}$$

$$-c\frac{\partial f}{\partial z} = \frac{\partial^2 f}{\partial z^2} - \frac{f}{\tau} + H(f-h)$$

$h>0$: $u=0$

$\tau>h$: $u=\tau$

$$\underline{-c\,\underbrace{U' = U''}_{=0} - \frac{U}{\tau} + H(U-h)}$$



I : $u'' + cu' - \frac{u}{\tau} = 0$

$\lambda^2 + c\lambda - \frac{1}{\tau} = 0$

$\Rightarrow \lambda = \dfrac{-c \pm \sqrt{c^2 + \frac{4}{\tau}}}{2}$

2 real soln: $\lambda_1 > 0$, $\lambda_2 < 0$

$u = Ae^{\lambda_1 z} + \underbrace{Be^{\lambda_2 z}}_{=0}$ $\quad B=0$

II : $u'' + cu' - \frac{u}{\tau} = -1$

$u_p = \tau$ $\qquad u = \underbrace{Ce^{\lambda_1 z}}_{=0} + De^{\lambda_2 z} + \tau$

$U(0)=h$ $\left\{ \begin{array}{l} A + \cancel{B} = h \\ \cancel{C} + D + \tau = h \end{array} \right.$ $\qquad A = h$, $\quad D = h-\tau$

continuity of $U'(0)$ $\{$ $A\lambda_1 + \cancel{B\lambda_2} = \cancel{C\lambda_1} + D\lambda_2$

$$h\left(\frac{-c+\sqrt{c^2+\frac{4}{\tau}}}{2}\right) = (h-\tau)\left(\frac{-c-\sqrt{c^2+\frac{4}{\tau}}}{2}\right)$$

$$h\sqrt{c^2+\frac{4}{\tau}} = (\tau-h)\sqrt{c^2+\frac{4}{\tau}} + \tau c$$

$$(2h-\tau)\sqrt{c^2+\frac{4}{\tau}} = \tau c$$

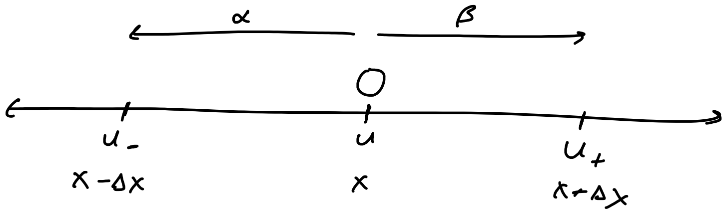
$$(2h-\tau)^2\left(c^2+\frac{4}{\tau}\right) = \tau^2c^2$$

$$\left((2h-\tau)^2 - \tau^2\right)c^2 = -(2h-\tau)^2\cdot\frac{4}{\tau}$$

$$c^2 = -\frac{4(2h-\tau)^2}{\tau(4h^2-4\tau h)}$$

$$= \frac{(2h-\tau)^2}{\tau h(\tau-h)} \qquad \tau>h$$

Continuing from week 8 Thu:



We have $\alpha+\beta=1$.

To model chemotaxis, suppose

$$\beta-\alpha = \underset{\text{constant}}{\varepsilon}\, w \qquad w = \frac{\partial v}{\partial x} = \text{gradient of chemical } v$$

Let's use the notation

$$\beta_- - \alpha_- = \varepsilon w_-,$$

$$\beta_+ - \alpha_+ = \varepsilon w_+.$$

The recursion for $u=u(x,t)$ is

$$u(t+\Delta t) = u - \underbrace{u(\alpha q_- + \beta q_+)}_{\substack{\text{particles moving} \\ \text{away from } x \text{ to} \\ x-\Delta x \text{ or } x+\Delta x}} + \underbrace{u_- \beta_- q}_{\substack{\text{particles} \\ \text{moving in} \\ \text{from left} \\ x-\Delta x}} + \underbrace{u_+ \alpha_+ q}_{\substack{\text{particles/cells} \\ \text{moving in} \\ \text{from right} \\ x+\Delta x}}$$

$$u(t+\Delta t) = u - \alpha u q_- - \beta u q_+ + \beta_- u_- q + \alpha_+ u_+ q$$

$$\boxed{\text{Note: from } \alpha+\beta=1 \text{ and } \beta-\alpha=\varepsilon w, \text{ we get } \alpha = \frac{1-\varepsilon w}{2},\ \beta = \frac{1+\varepsilon w}{2}}$$

$$= u - \frac{1-\varepsilon w}{2} u q_- - \frac{1+\varepsilon w}{2} u q_+ + \frac{1+\varepsilon w_-}{2} u_- q + \frac{1-\varepsilon w_+}{2} u_+ q$$

Rearranging, we get

$$2(u(t+\Delta t) - u) = -q_- u + \varepsilon q_- u w - q_+ u - \varepsilon q_+ u w + q u_- + \varepsilon q u_- w_- + q u_+ - \varepsilon q u_+ w_+$$

$$= \cdots = q u_+ - 2qu + q u_- - q_+ u + 2qu - q_- u - \varepsilon(q u_+ w_+ - q u_- w_- + q_+ u w - q_- u w)$$

Dividing by $2\Delta t$, we get

$$\frac{u(t+\Delta t)-u}{\Delta t} = \left(\frac{\Delta x^2}{2\Delta t}\right) q \cdot \frac{u_+ - 2u + u_-}{\Delta x^2} - \left(\frac{\Delta x^2}{2\Delta t}\right) \frac{q_+ + 2q - q_-}{\Delta x^2} \cdot u - \frac{\varepsilon \Delta x}{\Delta t}\left(q \cdot \frac{u_+ w_+ - u_- w_-}{2\Delta x} + \frac{q_+ - q_-}{2\Delta x} \cdot u w\right)$$

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Define ε so that

$$\chi = \lim_{\Delta x, \Delta t \to 0} \frac{\varepsilon \Delta x}{\Delta t} \quad \text{exists}$$

and let

$$D_u = \lim_{\Delta x, \Delta t \to 0} \frac{\Delta x^2}{2\Delta t} .$$

If you Taylor expand, take limits, remove higher order terms, you get

$$u_t = D_u\, q\, u_{xx} - D_u\, q_{xx}\, u - \chi\left(q \cdot (uw)_x + q_x\, uw\right)$$

$$= \left[D_u\left(q(u) - q'(u)u\right)u_x - \chi\left(q(u)\, u\, v_x\right)\right]_x$$

flux form