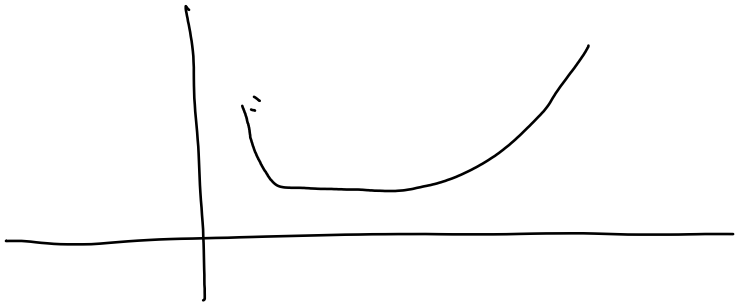
Thursday, August 10, 2017 9:56 AM

Discussion question 1

What is the shape of the Siler function?



Thursday, August 10, 2017 10:38 AM

u = prob of death

$1-\mu(x)$ = prob Survival

$$A_N = \int_{\text{DIS}}^{\infty} (1-\mu(x)) \cdot x \, dx$$

$$\frac{dn_c}{dt} = -\mu(c+t) n_c$$

$$\frac{dn_c}{n_c} = -\mu(c+t)\, dt$$

$$\ln n_c = -\int_0^t \mu(c+\tau)\, d\tau$$

$$n_c = e^{-\int \cdots}$$

Calculation of life expectancy from age 0:

$\mu(x)$ = mortality rate at age x

Let $f(x)$ = probability density function of individuals that die at age x.

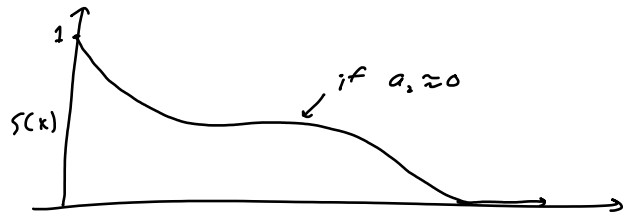
Then, life expectancy $= \int_0^\infty x f(x)\,dx$

Let

$S(x)$ = survival function

= prob. that an individual survives to at least age x

$= e^{-\int_0^x \mu(y)\,dy}$



Then, the prob that an individual dies between ages 0 and a is

$$\int_0^a f(x)\,dx$$

and is

$$S(0) - S(a) = 1 - S(a)$$

$$\Rightarrow \int_0^a f(x)\,dx = 1 - S(a) \Rightarrow f(a) = -\frac{dS(a)}{da}$$

So, life expectancy at age 0 is

$$\int_0^\infty x f(x)\,dx = -x\,S(x)\Big|_0^\infty + \int_0^\infty S(x)\,dx$$

$$= \int_0^\infty S(x)\,dx$$