Presentation Schedule

Tuesday, October 03, 2017 10:00 AM

Thu 5 Oct

Daniel
Kyle
Kate

Tue 10 Oct

Angus
Zach
Mitch
Ruben

Thu 12 Oct

Youstina
Anthia
Andy

Tue 17 Oct

Tim
Madeleine
Yuhuang
Sean

Thu 19 Oct

Alex
Cecilia
Sid

Tue 24 Oct

back up

$u_t = u_{xx} + u(u-\delta)(1-u)$

$U(z) = u(x,t),\ z = x - ct$

$u_t = \frac{dU}{dz}\frac{\partial z}{\partial t} = -cU_z$

$u_{xx} = U_{zz}$

$$-c\frac{dU}{dz} = \frac{d^2U}{dz^2} + U(U-\delta)(1-U)$$

$s = -\frac{dU}{dz}$

$\frac{dU}{dz} = -s \longrightarrow s = 0$

$\frac{ds}{dz} = -cs + U(U-\delta)(1-U)$

$(\bar{U}, \bar{s}) = (0,0),\ (\delta, 0),\ (1,0)$

sad stable sad

at $(1,0)$ $J = \begin{pmatrix} 0 & -1 \\ -1+\delta & -c \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -(1-\delta) & -c \end{pmatrix}$

$\det(J - \lambda I) = \lambda(c+\lambda) - (1-\delta)$ saddle

$\lambda = \frac{-c \pm \sqrt{c^2 + 4(1-\delta)}}{2}$

$J = \begin{pmatrix} 0 & -1 \\ [-3U^2 + 2U(1+\delta) - \delta] & -c \end{pmatrix}$

at $(0,0)$ $J = \begin{pmatrix} 0 & -1 \\ -\delta & -c \end{pmatrix}$ $\boxed{\text{saddle}}$

$\lambda = \frac{-c \pm \sqrt{c^2 + 4\delta}}{2}$

at $(\delta, 0)$

$J = \begin{pmatrix} 0 & -1 \\ -3\delta^2 + 2\delta + 2\delta^2 - \delta & -c \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -\delta^2 + \delta & -c \end{pmatrix}$

$\det(J - \lambda I) = \lambda(c+\lambda) + (\delta^2 - \delta)$ $0 < \delta < 1$

$\lambda = \frac{-c \pm \sqrt{c^2 - 4(\delta^2 - \delta)}}{2}$ both negative

$\Rightarrow (\delta, 0)$ stable

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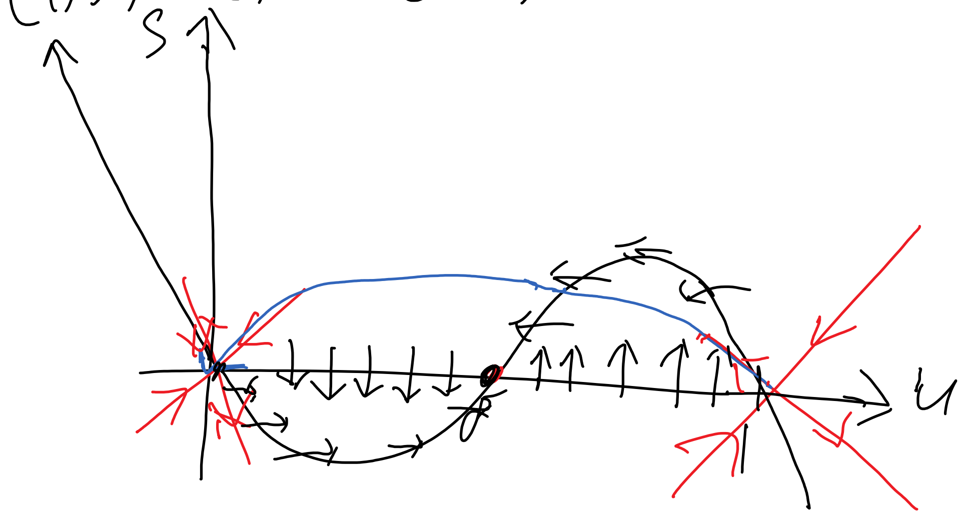
$(\bar{u}, \bar{s}) = (0,0), (\delta, 0), (1,0)$



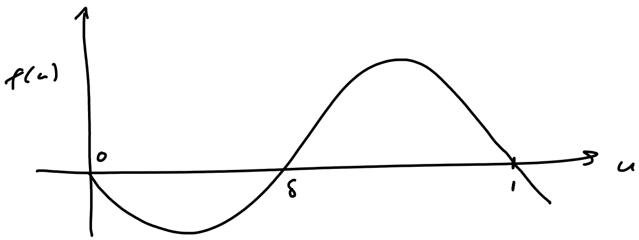
$$\frac{du}{dz} = -s$$

$$\frac{ds}{dz} = -cs + u(u-\delta)(1-u)$$

$$cs = u(u-\delta)(1-u)$$

We started with

$$U_t = u_{xx} + \underbrace{u(u-\delta)(1-u)}_{f(u)}$$



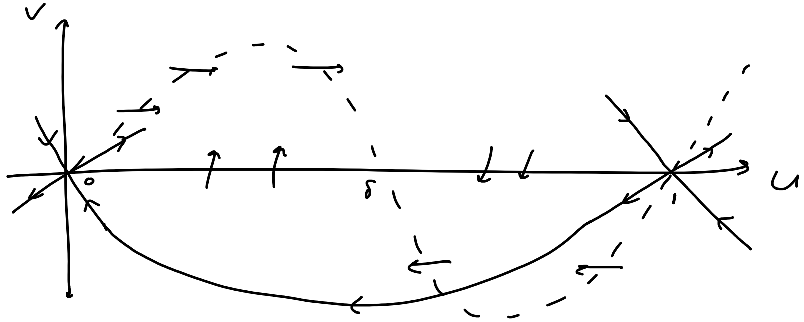
A growth rate of cubic shape $f(u)$ is called an Allee effect.

If a population falls below a threshold δ, it starts to die off. If it's greater than δ, it grows to capacity.

In wave coordinates, $U(z) = u(x,t)$ for $z = x - ct$, we get

$$U'' = -cU' - f(U).$$

Let $V = U'$, so
$$\begin{cases} U' = V \\ V' = -cV - f(U) \end{cases}$$

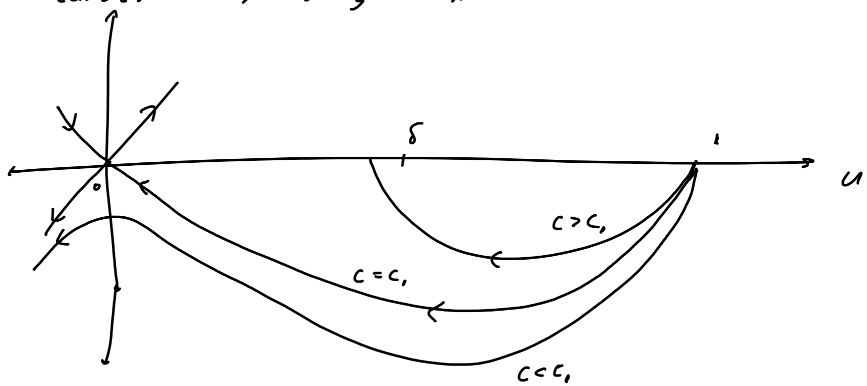


We will show that there is a unique wave speed that corresponds to a travelling wave from 1 to 0.

At any point (U_0, V_0), the curve through (U_0, V_0) has slope

$$\frac{-cV_0 - f(U_0)}{V_0} = -c - \frac{f(U_0)}{V_0}.$$

This slope is monotonically decreasing with respect to c. So, for increasing c, the curves originating from $(1,0)$ look like



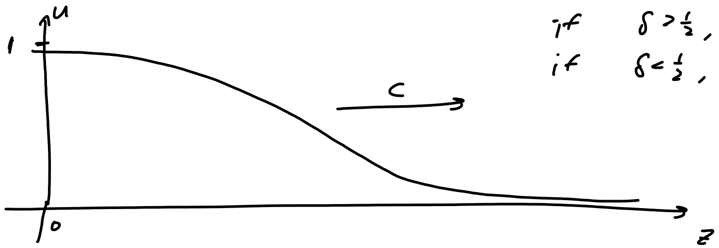
Hence, there is exactly one wave speed c that admits a solution such that $U(z) \to 1$ as $z \to -\infty$ and $U(z) \to 0$ as $z \to \infty$.

FYI, this trajectory has exact solution

$$U(z) = \left(1 + \exp\left(z/\sqrt{2}\right)\right)^{-1}$$

with wave speed $c = \sqrt{2}\left(\frac{1}{2} - \delta\right)$.

You can check this.



if $\delta > \frac{1}{2}$, $c < 0$

if $\delta < \frac{1}{2}$, $c > 0$