1. Hubert keeps five varieties (A, B, C, D, E) of snakes in boxes in his apartment. Some varieties attack other varieties, and can’t be kept together. In the table, an asterisk indicates that varieties can’t be kept together. What is the minimum number of boxes needed?

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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</thead>
<tbody>
<tr>
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</tbody>
</table>

2. Determine the number of ways in which each of the following graphs can be properly coloured, given $\lambda$ different colours.

(i) The complete graph $K_6$.

(ii) The star graph $K_{1,5}$.

(iii) The linear graph $L_6$.

3. (i) Find the chromatic polynomials of each of the six connected simple graphs on four vertices.

(ii) Verify that each of the polynomials in (i) has the form $\lambda^4 - e\lambda^3 + a\lambda^2 - b\lambda$ where $e$ is the number of edges and $a$ and $b$ are positive constants.

4. Find the chromatic polynomials of $K_{1,n}$, $K_{2,n}$ and $K_{3,n}$.

5. State two reduction formulas for chromatic polynomials. Use whichever seems appropriate to calculate the chromatic polynomial for each of the two given graphs. Also determine the chromatic number of each graph.

6. Find the chromatic polynomial of $C_5$, the cycle with 5 vertices.

7. Explain why the chromatic polynomial $P_G(\lambda)$ of a planar graph $G$ cannot contain a term $(\lambda - k)$ for any $k \geq 4$.

8. Find the chromatic index (or edge-chromatic number) of the graph $G$, where $G$ is:

(a) \[ \begin{array}{c}
    a \\
    \includegraphics{graph1.png} \\
    d \\
    c \\
    b
\end{array} \]

(b) \[ \begin{array}{c}
    a \\
    \includegraphics{graph2.png} \\
    d \\
    c \\
    b
\end{array} \]

9. Find the chromatic index of the cube, and of the octahedron.