Tutorial Week 1 (24-28 July)

(You need a copy of tables III and IV of p574-575)

1. Let $Z \sim \mathcal{N}(0, 1)$.
   - (i) $P(-1.72 < Z < 0.52)$ and $P(|Z| > 1.96)$.
   - (ii) Find the values of $a$, $b$, and $c$ such that $P(Z \leq a) = 0.7291$, $P(Z > b) = 0.10$ and $P(|Z| < c) = 0.90$.

2. If $X \sim \mathcal{N}(10, 16)$, find $P(X > 12)$, $P(X \leq 7)$, and $k$ such that $P(X \leq k) = 0.2$.

3. $X_1, X_2, \ldots, X_{16}$ represents a random sample from a normal distribution with mean $\mu = 20$ and a standard deviation, $\sigma = 4$. Explain why the distribution of $\bar{X} - 20$ is standard normal. Hence find the probability, $P(\bar{X} \geq 22)$.

4. Let $X_1, X_2, \ldots, X_{10}$ be a random sample of size 10 from an $\mathcal{N}(15, 2.53^2)$ distribution. Show that the probability, $P(\sum_{i=1}^{10} X_i \leq 158)$ is close to 0.8413.

5. Find
   - (i) $P(t_3 < 2.353)$ and $P(|t_{11}| > 2.718)$.
   - (ii) Find the values of $a$, $b$, and $c$ such that $P(t_{20} \leq a) = 0.10$, $P(|t_{16}| > b) = 0.05$ and $P(t_{10} > c) = 0.90$.

6. $X_1, X_2, \ldots, X_n$ ($n$ is small) represents a random sample from a normal distribution with a known mean $\mu$ and unknown standard deviation $\sigma$. State the distribution of $T = \sqrt{n} \frac{\bar{X} - \mu}{S}$, where $S$ is the sample standard deviation.

7. A random sample of size 25 from a normal population with mean, $\mu = 50$ is obtained. It gave the sample mean of 48 and the sample standard deviation of 4. Calculate a value of $T$, $T_{\text{obs}} = \sqrt{\frac{n}{s}} \frac{\bar{X} - \mu}{s}$, as given in Q6. What is the df, $\nu$ associated with this $t$ statistic? Find $P(t_{\nu} < T_{\text{obs}})$.

PTO for the Computer Exercise

In this semester we will learn how to use the statistical package R to perform calculations, simulations and carry out estimation and hypothesis testing related to this course. This R is freely available on internet.
1. Start R and complete the practice sheet.

2. Use appropriate R functions (see notes below) to check your answers in tutorial Q1 Q2 and Q5.

Notes

* Instead of using table III, one can use R to find probabilities and quantiles related to normal distributions as follows:

  \[ \text{pnorm}(x, \text{mean} = \mu, \text{sd} = \sigma) \] gives the probability for a given \( x \) such that \( P(X \leq x) \), where \( X \sim N(\mu, \sigma^2) \).

  \[ \text{qnorm}(p, \text{mean} = \mu, \text{sd} = \sigma) \] gives the value of \( x \) for a given \( p \) such that \( P(X \leq x) = p \), where \( X \sim N(\mu, \sigma^2) \).

Note: When \( Z \sim N(0, 1) \) the corresponding values are given by \( \text{pnorm}(z) \) and \( \text{qnorm}(p) \).

Practice Problem 1:

Suppose that \( X \sim N(75, 100) \). Find \( P(X < 90) \), \( P(80 < X < 90) \) and \( c \) such that \( P(X < c) = 0.93 \).

Steps in R

\[
\begin{align*}
> \text{pnorm}(90, \text{mean} = 75, \text{sd} = 10) \\
[1] 0.9331928 \\
\end{align*}
\]

\[
\begin{align*}
> \text{pnorm}(90, \text{mean} = 75, \text{sd} = 10) - \text{pnorm}(80, \text{mean} = 75, \text{sd} = 10) \\
[1] 0.2417303 \\
\end{align*}
\]

\[
\begin{align*}
> \text{qnorm}(0.93, \text{mean} = 75, \text{sd} = 10) \\
[1] 89.75791 \\
\end{align*}
\]

* Instead of using table IV, one can use the SPLUS to find probabilities and quantiles related to t-distributions.

  \[ \text{pt}(x, \nu) \] gives the probability for given \( x \) such that \( P(t_\nu \leq x) \)

  \[ \text{qt}(p, \nu) \] gives the value of \( x \) for given \( p \) such that \( P(t_\nu \leq x) = p \),

where \( \nu \) is the df.

Practice Problem 2:

Find \( P(t_{15} < 1.753) \), \( P(2.1 < t_{10} < 3.0) \) and \( c \) such \( P(t_8 < c) = 0.93 \).

Steps in R

\[
\begin{align*}
> \text{pt}(1.753, 15) \\
[1] 0.9499956 \\
\end{align*}
\]

\[
\begin{align*}
> \text{pt}(3, 10) - \text{pt}(2.1, 10) \\
[1] 0.02436679 \\
\end{align*}
\]

\[
\begin{align*}
> \text{qt}(0.93, 8) \\
[1] 1.638266 \\
\end{align*}
\]