Tutorial Week 11 (9/10-13/10)

This week we’ll look at One-Way design problems (ANOVA: Analysis of Variance). Please refer Chapter 15 of the textbook for details and notation.

Let \( SST = \sum_{i=1}^{a} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2 = \sum_{i=1}^{a} \sum_{j=1}^{n_i} x_{ij}^2 - \frac{T_i^2}{n_i} \), where \( n_i \) is the number of observations in the \( i^{th} \) sample, \( i = 1, 2, \cdots, a \) and \( T_i = \sum_{i=1}^{n_i} x_{ij} \) is the grand total of all \( N = n_1 + n_2 + \cdots + n_a \) observations. Clearly, \( SST \) is a measure of the total variance of the combined data. In one-way ANOVA, we partition \( SST \) into two components \( SSE \) (error or within sample variation) and \( SS(Tr) \) (treatment or between sample variation) and therefore

\[
SST = SS(Tr) + SSE,
\]

where \( SS(Tr) = \sum_{i=1}^{a} \frac{T_i^2}{n_i} - \frac{T^2}{N} \).

**Tutorial Problems**

1. Do Q15.16 (p513)

2. Do Q15.20 (p513)

3. The following partially completed ANOVA table reports the information on an experiment to measure the effect of smoking on heart rate. Six of each of non-smokers, light smokers, moderate smokers and heavy smokers undertook sustained physical exercise and their heart rates were measured after resting for three minutes.

<table>
<thead>
<tr>
<th>ANOVA Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
</tr>
<tr>
<td>Treatment</td>
</tr>
<tr>
<td>Error</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

(a) Fill the missing entries * of the above table.

(b) Is there any significant effect of smoking on heart rate? Justify.

**Extra Practice Problems**

1. Do Q15.17 (p513)

2. Do Q15.19 (p513)

PTO for the Computer Exercise
1. Let us consider an experimental study of drugs to relieve itching. Five drugs (N3-N7) were compared to a placebo (N2) and no drugs (N1) with 10 volunteer male subjects aged 20-30. This data is available in R with the name itch.

(a) Inspect the data by typing `itch`.

(b) Set a multiple graph window with 1 row and 2 columns (Using `par` or `Graph`)

(c) Obtain side by side boxplot of the data.

(d) Obtain a normal quantile plot of the combined residuals from the groups N1-N7 by using the following steps:
   
i. Create a group mean vector, say `g.mean`, by using `apply`. Hint:
      > g.mean=apply(itch,2,mean)
      > g.mean
   
   ii. Create a matrix, say `g.mat`, by using `matrix()`, in which the matrix matches to the data matrix "itch" and has the same group mean at each column. Hint:
      > g.mat=matrix(rep(g.mean,10),nr=10,byrow=T)
      > g.mat
   
   iii. Find the residual matrix of the data using
      > residual=itch-g.mat

and then perform a normal q-q-plot of the combined residuals.

(e) Comment on whether or not the data appears to satisfy the assumptions for an analysis of variance.

(f) Prepare a One-Way ANOVA table for this data. Hint:
   
   > itch=as.vector(itch)
   > fac=factor(rep(letters[1:7],c(rep(10,7))))
   > itch.g=data.frame(fac,itch)
   > aov.itch=aov(itch~fac,itch.g)
   > summary(aov.itch)

(There are a few alternative ways available to do this.)

(g) Comment on the results in (f). Test the null hypothesis of equality of means.

Extra Practice Problems

1. Do Q15.17 and Q15.19 (p513) using R. Check your answers.