Tutorial Week 7 (4/9-8/9)

This week we study two nonparametric tests

• The sign test: This is an alternative test to the one-sample t test.

• The Wilcoxon signed-rank test: This involves about the population median, but unlike the sign test it assumes that the population is continuous and symmetric (although not necessarily normal).

Statistics: Let $T^+$ be the sum of the ranks assigned to the positive differences, $T^-$ be the sum of the ranks assigned to the negative differences and $T = \min(T^+, T^-)$.

Critical Regions: The following table summarizes the critical values for a test of size $\alpha$:

<table>
<thead>
<tr>
<th>$H_1$</th>
<th>Reject $H_0$ if</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\mu} \neq \mu_0$</td>
<td>$T \leq T_\alpha$</td>
</tr>
<tr>
<td>$\hat{\mu} &gt; \mu_0$</td>
<td>$T^- \leq T_{2\alpha}$</td>
</tr>
<tr>
<td>$\hat{\mu} &lt; \mu_0$</td>
<td>$T^+ \leq T_{2\alpha}$</td>
</tr>
</tbody>
</table>

The mean and variance of $T^+$: When there are no ties on the data, it can be shown that $E(T^+) = \frac{1}{2}(1 + \cdots + n) = \frac{n(n+1)}{4}$ and $\text{var}(T^+) = \frac{1}{4}(1^2 + \cdots + n^2) = \frac{n(n+1)(2n+1)}{24}$.

Tutorial Questions

1. Q16.16 (p 544)
2. Q16.17 (p 544)
3. Q16.22 (p 545)

Extra Practice Problems

1. Q16.18 (p 544)

2. The following data gives the scores of a particular test of 14 students:
   Scores:51, 53, 43, 36, 55, 55, 39, 43, 45, 27, 21, 26, 22, 43.

It is known from the past that the mean score is 28. Test this null hypothesis (with a two-sided alternative) using the Wilcoxon signed-rank test. Compare your answer using the normal approximation.

Ans: $T = 10$ and reject $H_0$ if $T \leq 21$. Your conclusion? $\mu = 52.5; \sigma^2 = 253.75$. Calculate $Z$ and draw your conclusion.

PTO for the Computer Exercise
Tests

- `wilcox.test(x)` (for one sample) or `wilcox.test(x, y)` (for two samples) can be used to test $H_0$ against $H_1$. (See overleaf)

- The following versions can be used in one-sample and paired sample tests:

  `wilcox.test(x, a = "", \mu = \mu_0, exact = \,, correct = \)`  
  `wilcox.test(x, y, a = "", \mu = \mu_0, paired = T, exact = \,, correct = \)`  

  `a=""` specifies the hypothesis being tested. There are three options available: ”two.sided” or ”g” or ”l”

  `exact=T` computes the exact p-value; `correct=T` uses the normal approximation with the correction of continuity.

Practice

Supose that $x = c(83, 90, 129, 70)$ and $y = c(86, 93, 136, 82)$.

`wilcox.test(x, , mu = 50, exact = T, correct = F)` gives the p-value for a two-sided alternative. An additional argument can be included as follows:

`wilcox.test(x, mu = 50, a = "g", exact = T, correct = F)` to test $H_1 : \mu > 50$.

`wilcox.test(x, y, mu = 0, paired = T)` to test $H_0 : \mu_1 = \mu_2$ against $H_0 : \mu_1 \neq \mu_2$.

Handin Problems

1. Do Q16.22 (p545) using R.
2. Do Extra Practice Problem 2 (see overleaf) using R.
3. The data in `amp` are the results of measurements made using two methods on 15 pairs of tablets to determine the dosage of ampicillin. Use the Wilcoxon sign-rank test to test the hypothesis that there is a systematic difference between the two methods.