SCHOOL OF MATHEMATICS AND STATISTICS

MATHEMATICAL STATISTICS, F07

STAT4 (TSA) - ADVANCED TIME SERIES ANALYSIS AND FORECASTING

— Semester 2, 2008 —

Assignment 1 - Due on 28 August

1.

(a) What is meant by saying the stochastic process $\{X_t\,;\ t \in\in Z\}$ is strictly stationary? If $\{X_t\,;\ t \in\in Z\}$ is a stationary Gaussian process, can you say that it is strictly stationary. Give reasons.

(b) Suppose that $\{X_t\,;\ t \in\in Z\}$ is generated by $X_t = Z_t - \theta_1 Z_{t-1} - \theta_2 Z_{t-2}$, where $Z_{-1}, Z_0, Z_1, \cdots$ are independent and identically distributed (i.i.d.) random variables, each with moment generating function (mgf) $E[\exp(\lambda Z_i)] = m(\lambda)$. Show that the joint mgf $E[\exp(\sum_{i=1}^{n} \lambda_i X_{i+h})]$ is invariant for all $h \in Z$.

Deduce that $\{X_t\,;\ t \in Z\}$ is strictly stationary.

(c) By considering the autocorrelation structure of $Y_{t+1} - Y_t$ or otherwise show that

$$\begin{aligned} Y_t &= S_t + e_t \\ S_{t+1} &= S_t + u_{t+1} \end{aligned}$$

is an equivalent representation of an $ARIMA(0, 1, 1)$. Find the corresponding moving average coefficient assuming both $\{e_t\}$ and $\{u_t\}$ are uncorrelated $WN(0, 1)$ processes.

2.

(a) Obtain the theoretical autocorrelation function of $\{Y_t\}$, where $Y_t = (1-B)(1-B^{12})X_t$ and $X_t = Z_t - \theta Z_{t-1}$. What is the forecast function $Y_t(\ell)$ for $\ell > 0$?

(b) Compute $\gamma_X(1) = \text{Cov}\,(X_t,\, X_{t+1})$ for the process $\{X_t;\ t \in\in Z\}$ given by

$$X_t = W_t \cos\frac{\pi t}{5} + W_{t-1} \sin\frac{\pi t}{5} + Z_t,$$

where $\{Z_t\}$ and $\{W_t\}$ are two independent sequences of independent random variables with zero mean and variance 1 . Deduce that $\{X_t\}$ is not stationary.

3.

(a) Write down the Yule-Walker equations for the stationary AR(2) process $\{X_t\}$

satisfying $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + Z_t$. Find ρ_1 and ρ_2 in terms of α_1 and α_2, where $\{\rho_\kappa\}$ is the autocorrelation function of $\{X_t\}$, and $\{Z_t\} \sim WN(0, \sigma^2)$. Given that ρ_κ can be expressed as $\rho_\kappa = A_1\, g_1^\kappa + A_2\, g_2^\kappa\,;\ \kappa \geq 0$, where g_1^{-1} and g_2^{-1} are the zeros of $1 - \alpha_1\,\theta - \alpha_2\,\theta^2$, and A_1 and A_2 are arbitrary constants, *find* the value of $|A_1 - A_2|$ when $\alpha_1 = -0.3$ and $\alpha_2 = 0.1$.

(b) Let $\{Z_t\}$ and $\{W_t\}$ be mutually independent $WN(0, \sigma^2)$ processes.

Suppose that the series $\{X_t\}$ defined by $X_t = \alpha_t Z_t + \beta_t Z_{t-1} + W_t - W_{t-1}$, where $\alpha_t = -\sin\frac{\pi(2t-1)}{6}$ and $\beta_t = \cos\frac{\pi(2t-1)}{6}$. Find the mean and the autocovariance function of X_t. Deduce that $\{X_t\}$ is stationary.

[Hint: $cos(A + B) = cosA\ cosB - sinA\ sinB$]