Fractal and aggregate tessellations

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Abstract

Voronoi tessellation is one of the simplest model for many systems employing subdivisions of space. Quite often, however, more realistic modelling calls for more complex tessellations that could take into consideration the observed irregularity of cells. Let $T_0, T_1, \ldots$ be a sequence of tessellations which cells are associated with nuclei. For each $T_0$ cell $C(x)$ with nucleus $x$ the aggregate cell $C_1(x)$ of level 1 is the union of those cells of $T_1$ which nuclei lie in $C(x)$. The second level aggregate cell $C_2(x)$ is the union of those $T_2$ cells which nuclei lie in $C_1(x)$, etc. Such aggregate tessellations appear naturally, for instance, in modelling of hierarchical telecommunications networks, where the cells $C_n$ represent the service zones of switches ($n + 1$) levels above in the hierarchical chain. Even if all $T_i$ are Voronoi tessellations, the aggregate tessellations’ cells may be empty, they are not, in general, convex nor connected nor contain their nuclei. We present results for aggregate tessellations that are based on stationary random tessellations, mainly Poisson-Voronoi ones. We find expressions for a typical aggregate cell’s coverage probability, give bounds on variation of its boundary and study conditions assuring existence of the limit fractal tessellation as $n$ grows to infinity. The talk will (hopefully) be assisted by a computer program PVAT downloadable from the author’s web-page, enabling construction and visualisation of aggregate tessellations.