1. The coefficient of $x^n$ in the Taylor series of the function $x^2 \sin 3x$ is: 0 if $n$ is even or if $n = 1$ and $(-1)^{k-1} \frac{3^{n-2}}{(n-2)!}$ if $n = 2k + 1 \geq 3$ is odd.

(a) True  (b) False

2. $\sum_{n=1}^{\infty} \frac{(-1)^n (\cos n + i \sin n)^n}{n^2 + 1}$ is conditionally convergent.

(a) True  (b) False

3. $\sum_{n=1}^{\infty} \left( \frac{4 - i}{\sqrt{17} + 1} \right)^n$ converges, but not absolutely.

(a) True  (b) False

4. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(3iz + 2)^n}{n^2 + 1}$ is $\frac{2}{3}$.

(a) True  (b) False

5. A convergent sequence of differentiable functions has a differentiable limit.

(a) True  (b) False

6. It is possible to rearrange the terms of a convergent series $\sum (-1)^n a_n$ in which $a_n > 0$ in such a way that the new series diverges.

(a) True  (b) False

7. The coefficient of $x^2$ in the Taylor series for $x \sin(x - 5x^2)$ about the point $x = 0$, is $\frac{1}{2!}$.

(a) True  (b) False
8. If \( \{a_n\} \) and \( \{b_n\} \) are convergent sequences, then \( \{a_n + ib_n\} \) is a convergent sequence.
   (a) True  (b) False

9. The alternating series \( \sum (-1)^n \frac{5^n}{n!} \) converges although the first 5 terms do not decrease.
   (a) True  (b) False

10. The series \( \sum (-1)^n \frac{n^3 + i}{n^3 + 3in + 1} \) is convergent.
    (a) True  (b) False

11. The series \( \sum (-1)^n \frac{1}{n \ln(n^2 - 1)} \) is conditionally convergent.
    (a) True  (b) False

12. A function \( f(x) = \sum a_n x^n \) defines a differentiable function with derivative
    \( f(x) = \sum_{n=1}^{\infty} na_n x^{n-1} \) at any point \( x \) with \( |x| < R \), where \( R \) is the radius of convergence of the series.
    (a) True  (b) False

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**Answers:**
1. True;
2. False; (It is absolutely convergent.)
3. False; (It is absolutely convergent.)
4. False; (It is \( \frac{1}{3} \).)
5. False; (For example the sequence of differentiable functions \( \{f_n(x)\} \), where \( f_n(x) = x^n \), for \( 0 \leq x \leq 1 \) has a discontinuous and so not differentiable limit.)
6. True; (The series \( 1 - \frac{1}{2} + \frac{1}{3} - \ldots \) can be rearranged to have a divergent sum. For example, there is an integer \( n_1 \) such that \( S_{n_1} = \sum_{k=1}^{n_1} \frac{1}{2k+1} > 2^1 \). Now insert the term \( -\frac{1}{2} \) and then insert enough odd terms so that \( S_{n_2} - \frac{1}{2} > 2^2 \). Then insert \( -\frac{1}{4} \) Then choose \( n_3 \) so that \( S_{n_3} - \frac{1}{2} - \frac{1}{4} > 2^3 \). And so on. In this way we will get a series whose partial sums are greater than \( 2^\ell \) for every \( \ell \).
7. False; (It is 1).
8. True; (Use the limit laws.)
9. True;
10. False; (The \( n \)-th term does not tend to zero as \( n \to \infty \).)
11. True; (The series is convergent by the AST.

It is not absolutely convergent because \( \ln(n^2 - 1) < \ln n^2 = 2 \ln n \) and so \( \frac{1}{n \ln(n^2 - 1)} > \frac{1}{2n \ln n} \). Hence \( \sum \frac{1}{n \ln(n^2 - 1)} \) diverges by comparison with the series \( \sum \frac{1}{2n \ln n} \), which diverges by the integral test.)
12. True.