Prove that the LUB axiom plus the axioms for Addition, Multiplication, Distributivity and Order imply the PMC axiom.

Recall that it is possible to derive any one of the axioms CPC, LUB and PMC from any other using the axioms A, M, CD and O.

State the Archimedean axiom for the natural numbers.

Show that the least upper bound axiom LUB implies the Archimedean axiom.

Explain why the axiom ARCH does not imply the axiom LUB, for example. (The rational numbers satisfy ARCH, while they do not have the property LUB.)

Define \( \lim_{x \to \infty} f(x) \) for real functions \( f \).

Show that if \( f(n) = a_n \), for each natural number \( n \), then \( \lim_{x \to \infty} f(x) = l \), \( \lim_{n \to \infty} a_n = l \).

Define \( \lim_{x \to a} f(x) \).

Show that \( \lim_{x \to a} f(x) \) exists for simple functions \( f \).

State the limit laws for functions.

Use the limit laws to decide if \( \lim_{x \to a} f(x) \) exists for simple functions \( f \).

Define continuity and use the limit laws to show that simple functions are or are not continuous.

Give examples of continuous and discontinuous functions.

Define differentiability of a function.

Give examples of differentiable and non-differentiable functions.

Show that if \( f \) is differentiable at \( a \) then \( f \) is continuous at \( a \), but that the converse is not true.