1. Write down two non-isomorphic rings different from \( \mathbb{Z}_{25} \) which have 25 elements. 
Answer (2 Marks)

2. Find two different primes which divide \( 7 + 9i \) in \( \mathbb{Z}[i] \).
Answer (2 marks)

3. Given that 181 is a prime and that \( \sqrt{-1} = 19 \in \mathbb{Z}_{181} \), apply one step of Fermat’s Method of Descent in the process of writing 181 as a sum of two squares.
Answer (2 Marks)

4. Calculate the length of the non-repeating block of the decimal expansion of \( \frac{1245}{2^7 \times 71 \times 10^62} \).
Answer.

5. Find the length of the periodic part of the decimal expansion of \( \frac{24}{31} \).
Answer.
6. Given that 5 is a generator (primitive root) modulo 73, write down $\sqrt{-1} \in \mathbb{Z}_{73}$ as powers of 5.

Answer

7. Every non-zero element of $\mathbb{Z}_5[x]_{(x+1)(x^2+x+1)}$ is a unit.

(a) True (b) False

8. There are exactly 4 elements $x \in \mathbb{Z}_5 \oplus \mathbb{Z}_7 \oplus \mathbb{Z}_{13}$ such that $x^2 = -1$.

(a) True (b) False

9. If $p$ is a prime of the form $10k + 1$, then there are exactly 5 elements $x \in \mathbb{Z}_p$ such that $x^5 = 1$.

(a) True (b) False
Answers:
1. \( \mathbb{Z}_5 \oplus \mathbb{Z}_5, \; \mathbb{Z}_5[x]_{x^2}, \; \mathbb{Z}_5[x]_{f(x)}, \) where \( f(x) \) is essentially any polynomial of degree 2 in \( \mathbb{Z}_5[x] \).
2. \( 7 + 9i = (1 + i)(2 - i)(3 + 2i) \) and since \( 1 + i, 2 - i \) and \( 3 + 2i \) have prime norm, they are all prime in \( \mathbb{Z}[i] \) and so any two of them will do.
3. 
\[
19^2 + 1^2 = 2.181 \\
1^2 + 1^2 = 2, \; \text{and so} \\
181.2^2 = (19^2 + 1^2)(1^2 + (-1)^2) \\
= (19.1 - 1(-1))^2 + (19.(-1) + 1.1)^2, \\
= 20^2 + 18^2, \; \text{and so} \\
181 = 10^2 + 9^2.
\]
4. 69
5. 15;
6. \( \pm 5^{18} \) or \( 5^{18} \) and \( 5^{54} \).
7. False; \( (x + 1) \) has no inverse in \( \mathbb{Z}_5[x]_{(x+1)(x^2+x+1)} \).
8. False: There is no element such that \( x^2 = -1 \in \mathbb{Z}_7 \) and so no element in \( \mathbb{Z}_5 \oplus \mathbb{Z}_7 \oplus \mathbb{Z}_{13} \).
9. True; (If \( g \) is a primitive root modulo \( p \) the elements are \( 1, g^{2k}, g^{4k}, g^{6k}, g^{8k} \))