MATH 3009, NUMBER THEORY, 2003, WEEK 6 SUMMARY

- Discuss the possibility of the existence or not of a solution of a system of congruences if the moduli $m_1, m_2, \ldots, m_n$ are not pairwise co-prime.

- Solve specific cases of the Chinese Remainder Theorem in $\mathbb{Z}$.

- The Chinese Remainder Theorem may or may not have a solutions when the moduli $m_1, m_2, \ldots, m_n$ are not pairwise co-prime. Solve specific cases and decide if there is no solution, in $\mathbb{Z}, \mathbb{Z}[i], \mathbb{Z}_p[x]$.

- Find units in $\mathbb{Z}[i]$ modulo $a + bi$, for specific Gaussian integers $a + bi$.

- Find units in $\mathbb{Z}_p[x]$ modulo $f(x)$, for specific polynomials $f(x) \in \mathbb{Z}_p[x]$.

- Show that any prime $p \equiv 3 \pmod{4}$ cannot be written as a sum of two squares. Hence show that primes $p \equiv 3 \pmod{4}$ in $\mathbb{Z}$ remain prime in $\mathbb{Z}[i]$.

- Show that a Gaussian integer $z$ with $N(z) = p$, a prime in $\mathbb{Z}$, is a prime in $\mathbb{Z}[i]$.

- Write specific Gaussian integers as a product of units and prime powers in $\mathbb{Z}[i]$. 