Assignment 2

“Curves”

Due Thursday, 18 August, at 16:00 in the assignment box for MATH3405. The box is located on Level 4 in the Mathematics (Priestley) building (67). It is number 035 in the brown lot of boxes (there are two lots, to find ours, turn right as you come from the stairs).

Please use a cover sheet!

Clearly state your assumptions and conclusions, and justify all steps in your work. Marks will be deducted for sloppy or incomplete working.

Q1 (Length)
Let $c \in \mathbb{R} \setminus \{0\}$, and define $\alpha : \mathbb{R} \rightarrow \mathbb{R}^2$ by
$$t \mapsto (e^{ct}\cos t, e^{ct}\sin t).$$

(a) Determine length $\alpha_{[a,b]}$ for any $a, b \in \mathbb{R}$ with $a \leq b$.
(b) Determine each of $\text{length}_{\mathbb{R}}(\alpha)$, $\text{length}_{(-\infty,a]}(\alpha)$ and $\text{length}_{[a,\infty)}(\alpha)$, where $a \in \mathbb{R}$ and length takes values in $[0, \infty]$.

Q2 (Angle)
Let $\alpha_i : I_i \rightarrow \mathbb{R}^n$, $i \in \{1, 2\}$, be two regular, continuously differentiable curves with $\alpha_1(t_1) = \alpha_2(t_2)$. The angle of intersection between $\alpha_1$ and $\alpha_2$ at $t_1$ and $t_2$ respectively is the angle $a$, which satisfies $0 \leq a \leq \pi$ and
$$\cos a = \frac{\alpha'_1(t_1) \cdot \alpha'_2(t_2)}{||\alpha'_1(t_1)|| \cdot ||\alpha'_2(t_2)||}.$$  

(a) Show that the angle of intersection is invariant under continuously differentiable ($C^1$) positive reparametrisations of $\alpha_1$ and $\alpha_2$.
(b) How does the angle change if one of the curves is changed by a $C^1$ positive and the other by a $C^1$ negative reparametrisation?
(c) How does the angle change if both curves are changed by $C^1$ negative reparametrisations?

Q3 (Curvature and torsion)
Let $\alpha : \mathbb{R} \rightarrow \mathbb{R}^3$ be defined by
$$\alpha(t) = (e^t \cos t, e^t \sin t, e^t).$$

Find the orthonormal frame $(T, N, B)$, curvature $\kappa$ and torsion $\tau$ as functions of $t$. 