Assignment 3

Your solutions should be submitted by the beginning of the lecture on
Tuesday, 4 October 2011.

Please attach a cover sheet!

Q1 Let $X$ be a normed space over $\Lambda$ and $Y$ be a closed subspace of finite co-dimension.
Let $f : X \rightarrow \Lambda$ be a linear functional with the property that $f|_Y : Y \rightarrow \Lambda$ is continuous.
Show that $f$ is continuous.

Q2 Let $X$ be a normed space over $\Lambda$, and $f : X \rightarrow \Lambda$ be a non-zero linear functional on $X$.
Show that the following are equivalent:
(a) $f \notin X^*$,
(b) $f(B(X)) = \Lambda$,
(c) $\ker f$ is dense in $X$.

Q3 Let $K$ be a subset of $l_p$, where $1 \leq p < \infty$. Show that the following are equivalent:
(a) $K$ is compact;
(b) $K$ is closed and for each $\varepsilon > 0$, there exist $y_1, \ldots , y_{n(\varepsilon)} \in K$, such that
$$K \subseteq \bigcup_{k=1}^{n(\varepsilon)} B(y_k; \varepsilon);$$
(c) $K$ is closed, bounded and for each $\varepsilon > 0$, there exists $m = m(\varepsilon)$, such that
$$\sum_{k=m+1}^{\infty} |x_k|^p < \varepsilon$$
for all $x = (x_k)_{k=1}^{\infty} \in K$. 