**Semester 1 First Assignment 2016**

This assignment comprises a total of 60 marks, and is worth 15% of the overall assessment. It should be completed, accompanied by a signed cover sheet, and a hardcopy handed in at the lecture on Wednesday 20 April. Acknowledge any sources or assistance. An electronic copy or scan should also be downloaded using Turnitin from the Blackboard portal.

1. Build a combined truth table for the following wffs, where $P$, $Q$, and $R$ are propositional variables:

   (a) $P \Rightarrow (Q \lor R)$

   (b) $(P \lor Q) \Rightarrow R$

   Use your tables to explain briefly why

   $$(P \lor Q) \Rightarrow R \models P \Rightarrow (Q \lor R),$$

   but

   $$P \Rightarrow (Q \lor R) \not\models (P \lor Q) \Rightarrow R.$$  

   (8 marks)

2. Use the rules of deduction in the Propositional Calculus (but avoiding derived rules) to find formal proofs for the following sequents:

   (a) $P, (P \land Q) \Rightarrow \neg R \vdash R \Rightarrow \neg Q$

   (b) $Q \Rightarrow (R \Rightarrow \neg Q) \vdash \neg (R \land Q)$

   (c) $(Q \Rightarrow R) \land (P \Rightarrow S) \vdash (P \lor Q) \Rightarrow (R \lor S)$

   (12 marks)

3. Use truth values to determine which one of the following wffs is a theorem (in the sense of always being true).

   (a) $\left((P \lor R) \land (Q \lor R)\right) \Rightarrow \left((P \land Q) \lor R\right)$

   (b) $\left((P \lor R) \land (Q \lor R)\right) \Rightarrow \left((P \lor Q) \land R\right)$

   For the one that isn’t a theorem, produce all counterexamples. For the one that is a theorem, provide a formal proof also using rules of deduction in the Propositional Calculus (but avoiding derived rules of deduction).

   (10 marks)
4. A *tilde-arrow-wff* is a well-formed formula that is built out of propositional variables and the logical connectives \( \sim, \Rightarrow \) and \( \Leftrightarrow \) only. For example

\[ W_1 \equiv ((\sim ((\sim P) \Leftrightarrow Q)) \Rightarrow ((\sim R) \Rightarrow (P \Leftrightarrow Q))) \]

is a tilde-arrow-wff. For part (a) below, each bracket and each propositional variable, including its subscript, counts as a symbol. We also include in the count of symbols the final brackets on the outside of the wff that are normally invisible in practice.

(a) Prove that the number of symbols of tilde-arrow-wffs can be 1, 4, 5 or any integer greater than or equal to 7. (We saw in lectures that the number of symbols of a tilde-arrow-wff cannot be 2, 3 or 6, and you do not need to prove this.)

(b) Use truth values to explain why \( W_1 \) is a theorem.

(c) Find a tilde-arrow-wff \( W_2 \) that has the following truth table (including a brief explanation how you found it):

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(d) Prove that all truth tables, in any number of propositional variables, arise as truth tables of tilde-arrow-wffs.

(15 marks)

5. Evaluate each of

\[ \frac{1}{3}, \frac{7}{8}, \frac{8}{7} \]

in \( \mathbb{Z}_{12} \) and \( \mathbb{Z}_{13} \), or explain briefly why the given fraction does not exist.

(6 marks)

6. Prove that the only integer solution to the equation

\[ x^2 - 3y^2 = 2z^2 \]

is \((x, y, z) = (0, 0, 0)\). 

(9 marks)