**The University of Sydney**

**MATH3066 Algebra and Logic**

Semester 1  **Week 6 Exercises**  2016

*Starred questions are suitable for students aiming for a credit or higher.*

**1.** Verify the following where $W_1$ and $W_2$ are wffs in the Propositional Calculus:
   
   (a) $W_1 \vdash W_2$ if and only if $\vdash W_1 \Rightarrow W_2$.
   
   (b) $W_1 \models W_2$ if and only if $\models W_1 \Rightarrow W_2$.

**2.** The following sequents were relied upon in our proof of completeness of the Propositional Calculus. Check that they hold using the ten rules of deduction.
   
   (a) $Q \vdash Q$
   
   (b) $Q \vdash \neg \neg Q$
   
   (c) $R, S \vdash R \land S$
   
   (d) $\neg R \vdash (\neg R) \lor (\neg S)$
   
   (e) $\neg R \vdash \neg (R \land S)$
   
   *(f) $(\neg R) \lor (\neg S) \vdash \neg (R \land S)$
   
   *(g) $(\neg R) \land (\neg S) \vdash \neg (R \lor S)$
   
   *(h) $\neg R \vdash R \Rightarrow S$
   
   *(i) $S \vdash R \Rightarrow S$
   
   *(j) $R, \neg S \vdash \neg (R \Rightarrow S)$

**3.** Today is Monday. What day will it be after 100 days have elapsed?

*4.** It has been predicted that exactly 100 hours from 9 a.m. this Monday a meteor will strike the earth. At what time and day will this event happen?

**5.** Evaluate the following when they exist in $\mathbb{Z}_7$, $\mathbb{Z}_8$ and $\mathbb{Z}_9$:
   
   $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{3}{5} \cdot \frac{5}{6}$

*6.** Let $x = d_k d_{k-1} \ldots d_2 d_1$ be the base 10 representation of an integer $x$ where $d_1, \ldots, d_k$ are digits drawn from 0, \ldots, 9. Explain why
   
   $x = d_1 + d_2 + \ldots + d_k \pmod{9}$
   
   so, also,
   
   $x = d_1 + d_2 + \ldots + d_k \pmod{3}$
   
   and
   
   $x = d_1 - d_2 + d_3 - d_4 + \ldots + (-1)^{k-1} d_k \pmod{11}$.
   
   Thus for example to check whether 57,711 is divisible by 9 or 3 we just add up the digits $5 + 7 + 7 + 1 + 1 = 21$. Since 21 is divisible by 3 but not by 9 the same holds for 57,711. To check whether 219,186,429 is divisible by 11 we find the alternating sum $9 - 2 + 4 - 6 + 8 - 1 + 9 - 1 + 2 = 22$. Since 22 is divisible by 11 so is 219,186,429.